The real interest rate channel *is* structural in contemporary New-Keynesian models: A Note*

Joshua Brault†
Carleton University
& Ottawa-Carleton GSE

Hashmat Khan‡
Carleton University
& Ottawa-Carleton GSE

October 18, 2019

Abstract

The monetary transmission mechanism in a New-Keynesian model with contemporary features is put to scrutiny. In contrast to Rupert and Šustek (2019), we find that the real interest rate channel *is* structural when the model contains empirically realistic frictions on the flow of investment. A monetary contraction (expansion) is always followed by an increase (decrease) in the real interest rate. The monetary transmission mechanism indeed operates through the real interest rate channel in this class of models, affecting both short- and long-run interest rates in the same direction.

Key words: New-Keynesian models, monetary transmission mechanism, real interest rate
JEL classification: E24, E32, E43

*We thank Sanjay K. Chugh (the Editor) and an anonymous referee for thoughtful suggestions. We also thank Jordi Galí for helpful comments.

†Department of Economics, D886 Loeb, 1125 Colonel By Drive, Carleton University, Ottawa, Canada. *E-mail:* joshua.brault@carleton.ca. Tel: +1.613.520.2600 (ext 3057).

‡Corresponding author. Department of Economics, C877 Loeb, 1125 Colonel By Drive, Carleton University, Ottawa, Canada. *E-mail:* hashmat.khan@carleton.ca. Tel: +1.613.520.2600 (ext 1561).

1 Introduction

How does monetary policy affect inflation and output? According to contemporary New Keynesian (NK) models that are widely used in academia and central banks, it is via the *the real interest rate channel*. An increase in the short-term nominal interest rate increases the real interest rate in the presence of sticky prices. Households and firms then reduce consumption and investment, respectively. As demand and output contract, inflation declines. In a recent provocative paper, Rupert and Šustek (2019) challenge this widely held view. They write:

> *The main message of this paper is that the transmission mechanism of monetary policy in New-Keynesian models does not operate through the real interest rate channel. Any consistency with the real interest rate channel is purely observational, not structural, due to a specific parameterization.* Rupert and Šustek (2019), p. 54.

Based on their analysis using an NK model with capital, they conclude that from a monetary policy perspective either current NK models present a misleading description of the monetary transmission mechanism or policy makers need to rethink the monetary transmission channel altogether.

In this note, we show that the properties highlighted in Rupert and Šustek (2019) rely on two specific features both of which are absent in contemporary NK models that are used for monetary policy analysis by academics (for example, the literature following Christiano, Eichenbaum and Evans (2005) and Smets and Wouters (2007)), and in Central Banks (for example, Brave et al. (2012), Del Negro et al. (2013), among many others). First is that in the frictionless setting, with smooth consumption, the analysis relies on an unrealistic response of investment to a monetary policy shock —investment deviates upwards of 50% from steady state after a 1% shock to the policy rate. Depending on the persistence of the monetary shock, the (ex-ante) real interest rate can increase, decrease, or remain unchanged, and in this sense the response is not structural. None of the contemporary NK models, however, have this feature. Second is that with capital adjustment costs, Rupert and Šustek (2019) show that the real interest rate channel arises only for sufficiently high capital adjustment costs. For low costs, the real interest rate moves in the opposite direction from the monetary shock. The real interest rate response is again not structural. None of the contemporary NK models, however, consider capital adjustment costs. Instead, these models have adjustment costs on the flow of investment, or Investment Adjustment Costs (IAC) as introduced by Christiano, Eichenbaum and Evans (2005) to match the empirical response of investment to a monetary policy shock.[1]

We illustrate that in an NK model with IAC the real interest rate channel *is* structural, contrary to the conclusions of Rupert and Šustek (2019). The real interest rate always has the same sign as that of the monetary shock. A monetary contraction raises the real rate whereas an expansion lowers it. This response of the real interest rate does not depend on the size of IAC or the degree of persistence in the monetary shock process. In this sense, the real interest rate channel in NK models *is* structural. Both consumption and investment adjust to the real interest rate. Hence, the monetary transmission mechanism indeed operates through the real interest rate channel in contemporary NK models. We also show that both short- and long-rates move in the same direction.

While we are not the first to emphasize the importance of adjustment costs for determining the sign of the real interest rate in response to a monetary policy shocks, we illustrate this point in a prototypical New-Keynesian model with investment adjustment costs which makes our results directly comparable to those in Rupert and Šustek (2019) and the contemporary literature. Previously, for example, Kimball (1995) derives analytical expressions for the real interest rate to display a liquidity effect in response to a permanent positive money supply

[1] The use of IAC in contemporary NK-DSGE models is ubiquitous.

shock in the presence of capital adjustment costs in a sticky price model. Basu and Kimball (2005) show that investment planning costs also generate a liquidity effect which is more in line with the data than capital adjustment costs. Woodford (2003) (p. 352) shows that a New-Keynesian model with firm-specific capital and capital adjustment costs can produce a liquidity effect. A recent discussion of the behavior of real interest rates and consumption dynamics in response to a total factor productivity shock within a medium scale NK-DSGEs is presented by L'Huillier and Yoo (2019).

In Section 2 we lay out an NK model with endogenous capital and IAC and illustrate the real interest rate channel. We consider different parameterizations of IAC and the persistence of monetary shock to support our main point. In Section 3 we conclude.

2 New-Keynesian model with capital

In this section we assess the New-Keynesian model with capital and investment adjustment costs. Since the model is nearly identical to the NK model in Rupert and Šustek (2019), we simply highlight the one modified equation. The law of motion for capital with IAC takes the following form,

$$K_{t+1} = (1-\delta)K_t + I_t\left[1 - \frac{\Omega}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right], \tag{1}$$

where K_t is the current capital stock, I_t is current gross investment, δ is the depreciation rate, and Ω is the IAC parameter. Under this adjustment costs specification, the more the gross growth rate of investment differs from one the less new capital is produced from a unit of investment. In this setup Ω governs the magnitude of the costs associated with IAC. We describe the equilibrium conditions of the model and the log-linearized equations in the

Appendix.

To minimize differences between the responses reported here and in Rupert and Šustek (2019), we use the same calibration of parameter values: $\beta = 0.99$, $\eta = 1$, $\theta = 0.7$, $\nu = 1.5$, $\delta = 0.025$, $\alpha = 0.3$, and $\varepsilon = 0.83$.[2] We calibrate the adjustment costs parameter to 5.48 which corresponds to the estimated value in Smets and Wouters (2007). However, industry level IAC estimates tend to be smaller (Groth and Khan 2010). We show that the conclusions regarding the real interest rate channel hold under smaller adjustment costs parameter specifications. The real interest rate is reported as percentage point deviation from steady state (i.e., $R_t - \bar{R}$) while consumption, investment, and output are reported in percentage deviation from steady state (i.e., $\frac{x_t - \bar{x}}{\bar{x}}$). Figure 1 displays the response of consumption, investment, the real interest rate, and output to a 1 percentage point shock to the monetary policy rule.

The impulse response functions show that consumption, investment, and output fall in response to a positive monetary policy shock. In contrast to the ambiguity in Rupert and Šustek (2019) regarding the response of the real interest rate, when the model has frictions on the flow of investment, the real interest rate always rises in response to a positive monetary policy shock. This result holds under both shock persistence specifications.[3] Experimenting with the model, we find that even with highly persistence shocks ($\rho = 0.999$) the real interest rate rises when IAC is present in the model.[4]

The IAC parameter determines the strength of the de-linkage between the real interest rate and the marginal product of capital. We document that the real interest rate channel is robust to lower IAC parameters. Figure 2 displays the response of consumption, investment,

[2] We explored the importance of consumption habits for the real interest rate channel. Since they did not impact the conclusions drawn, we set $\varepsilon_C = 0$, as in Rupert and Šustek (2019).

[3] We do not report impulse responses for the case where $\rho = 0$, but the real interest rate also rises in this case.

[4] In an NK model without capital, the real interest rate always rises after a positive monetary shock (see, for example, Galí (2015), and also noted in Rupert and Šustek (2019)).

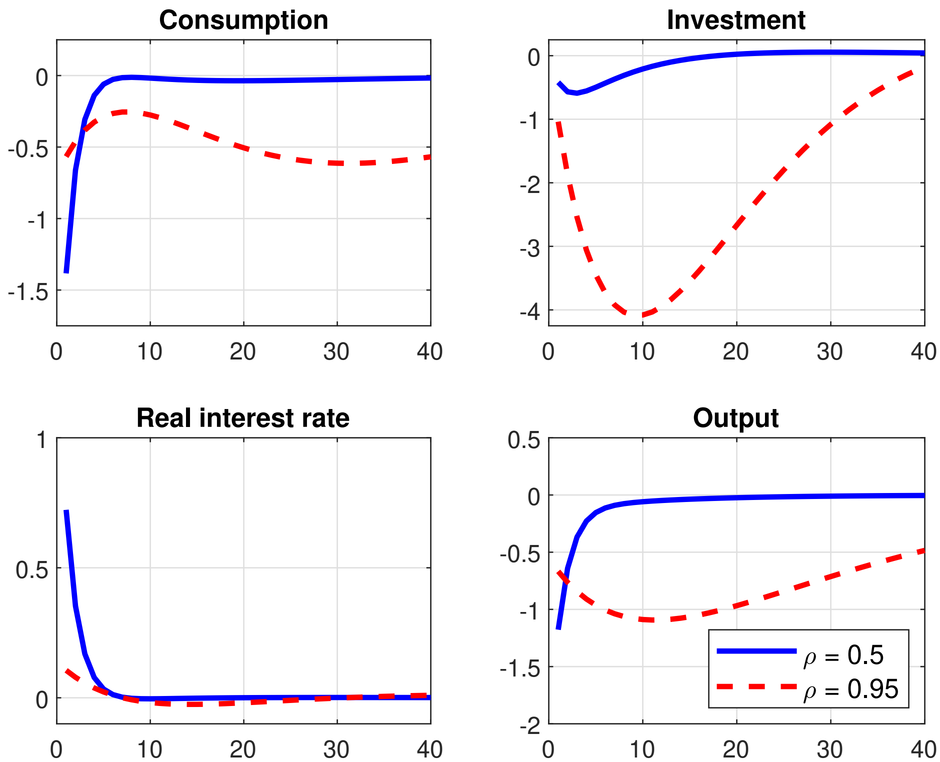

Figure 1: Impulse response functions to a monetary policy shock with investment adjustment costs

Notes: Investment adjustment costs parameter, Ω, is calibrated to 5.48. ρ is the persistence of the monetary policy shock.

the real interest rate, and output to a monetary policy shock when $\Omega = 2.5$ — a much smaller IAC parameter than typically estimated in the DSGE literature.[5] Naturally as the adjustment costs associated with the flow of investment fall, the response of investment to a monetary policy shock becomes larger. However, the real interest rate always rises.

In the absence of costs associated with the flow of investment, capital is extremely sensitive to changes in the real interest rate which lead to large changes in investment. To illustrate this point, Figure 3 displays the response of consumption, investment, the real interest rate, and output to a 1 percentage point increase in ξ_t when investment is frictionless

[5]For reference, Christiano et al. (2014) find an estimate of $\Omega = 10.78$.

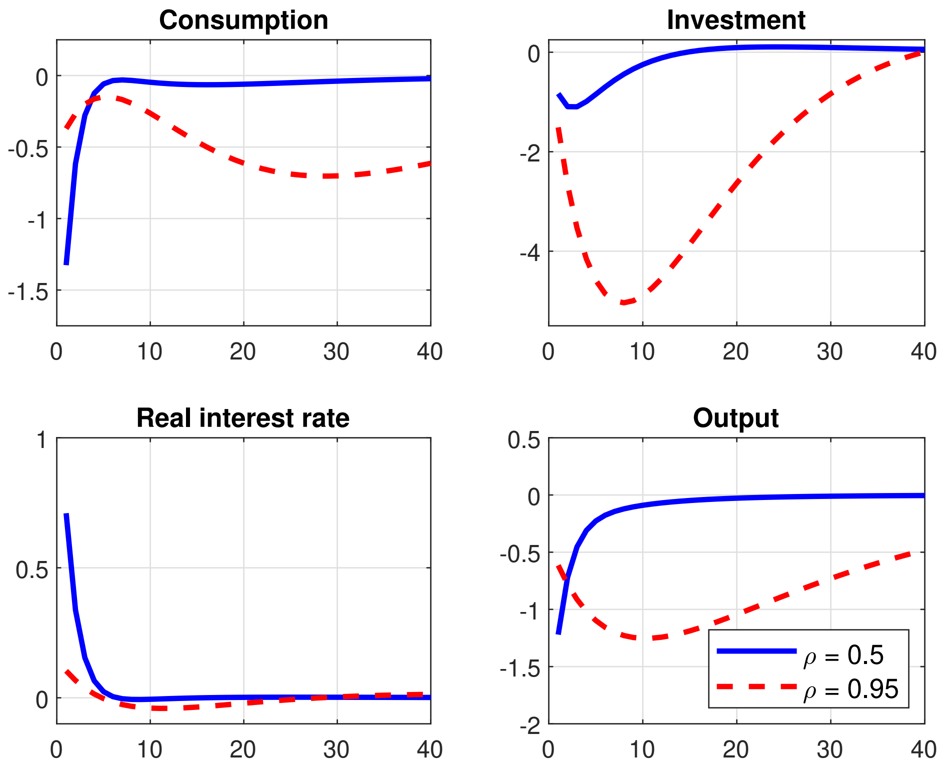

Figure 2: IMPULSE RESPONSE FUNCTIONS TO A MONETARY POLICY SHOCK WITH (SMALL) INVESTMENT ADJUSTMENT COSTS

Notes: Investment adjustment costs parameter, Ω, is calibrated to 2.5. ρ is the persistence of the monetary policy shock.

$(\Omega = 0)$. The impulse response functions reported here are identical to those in Figures 1, 3 and 4 in Rupert and Šustek (2019). Two points are worth emphasizing: First, the response of investment to a 1 percentage point shock to the monetary policy rate is unrealistic. Investment deviates from steady state by 13-53% depending upon the persistence in the shock.[6] Second, the response of the real interest rate is ambiguous. When shock persistence is low, the real interest rate rises. But even moderate amounts of persistence lead to a fall in the real interest rate. While the emphasis in their exercise is on the qualitative features of the model, our point is that drawing implications based on these model properties, as they do,

[6]The response of investment to a monetary shock is not shown in Rupert and Šustek (2019).

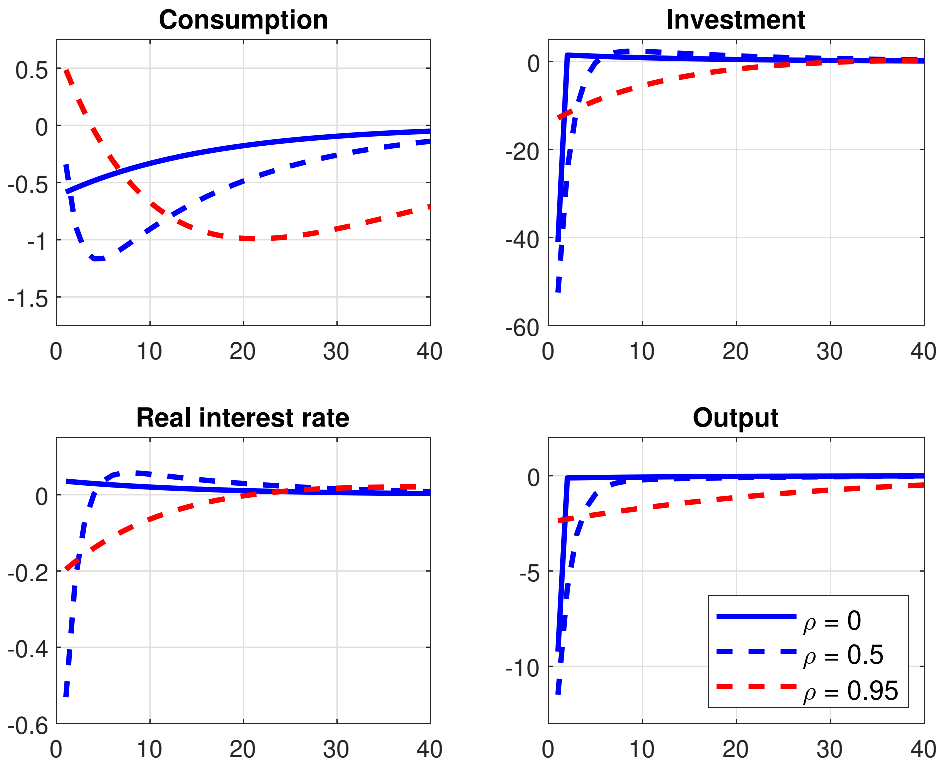

Figure 3: Impulse response functions to a monetary policy shock in Rupert and Šustek (2019)

Notes: Investment adjustment costs parameter, Ω, is calibrated to 0. ρ is the persistence of the monetary policy shock.

is problematic.

Long-run real interest rates and investment

Capital is a long-lived asset. Current investment decisions depend not only on the current real rate of return, but also the full path of expected real rates of return.[7] Falling short-run real interest rates and investment can be consistent with the real interest rate channel if the long-run real interest rate rises.[8] We investigate this point under no adjustment costs, CAC,

[7] We thank an anonymous referee for raising this point.

[8] In the absence of adjustment costs the real return on bonds is equal to the real return on capital.

and IAC.

To define long-run real interest rate, we follow the same approach as in Rupert and Šustek (2019)). By iterating the log-linearized Euler equation for bonds forward (and imposing convergence to the steady state in the absence of shocks) we obtain,

$$-\hat{C}_t = E_t \sum_{t=0}^{\infty} \hat{R}_{t+j} \equiv R_t^{\ell} \tag{2}$$

where R_t^{ℓ} is the long-run real interest rate.

Figure 4 reports the response of investment and the long-run real interest rate in response to a positive one percentage point monetary policy shock under alternative adjustment costs scenarios and shock persistence parameterizations.

The left column in Figure 4 shows that the long-run real interest rate rises even when the short-run real rate falls in the no adjustment cost case after a monetary contraction, with low shock persistence. This degree of persistence is also empirically relevant. For example, the posterior mode of the monetary shock persistence parameter in Smets and Wouters (2007) (Table 1B) is 0.12, indicating that monetary shock is of transitory nature. When the shock persistence is high (when $\rho = 0.85$), the long-run real interest rate falls (right column). But, as mentioned above, this calibration is not supported in U.S. data.

With respect to long-run real interest rates, it is worth emphasizing two points which favor the IAC version of the model. First, with IAC, the long-run real interest rate rises under any persistence parameterization. In contrast, in the no adjustment costs case, the long run real interest rate falls if the shock is very persistent. While CAC leads to a rise in the long-run real interest under moderate and high shock persistence, the short-run real interest rate falls (as emphasized by Rupert and Šustek (2019) in their Figure 8). Only the IAC version of the model leads to a rise in both the short run and long run real interest rates

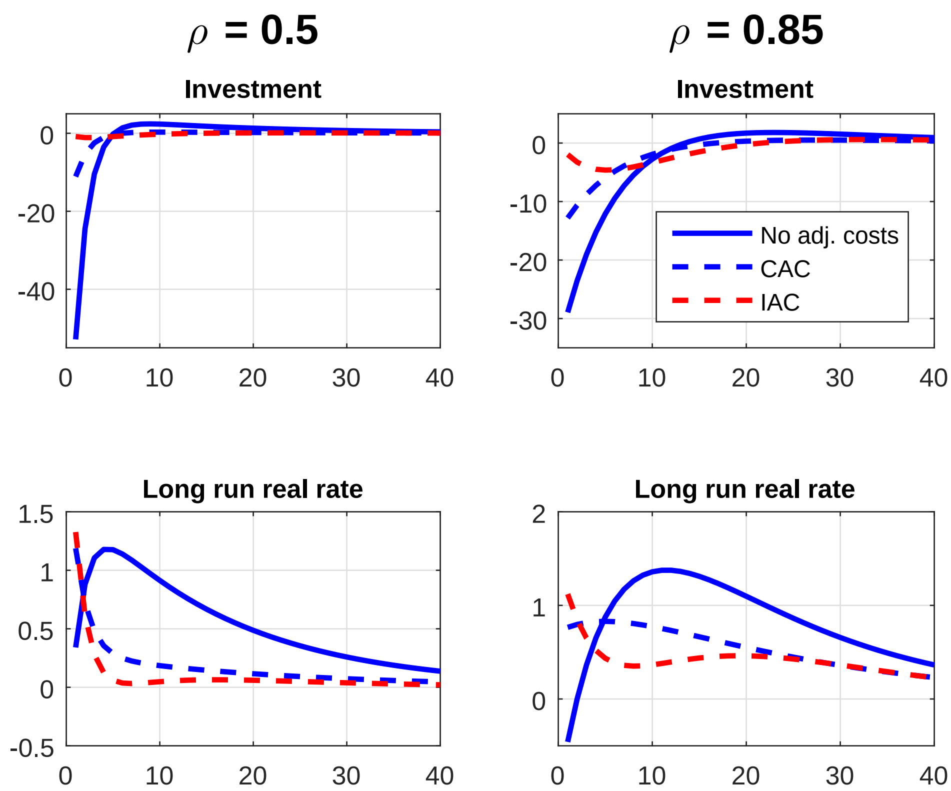

Figure 4: Long-run real interest rate and investment in response to a monetary policy shock under alternative adjustment costs

Notes: The left column plots the response of investment and the long-run real interest rate when the shock persistence, ρ, is 0.5. The right column plots the responses when $\rho = 0.85$. For capital adjustment costs we use a adjustment parameter of 0.5 and for investment adjustment costs we use a parameter of 2.5.

under any shock persistence calibrations.

Second, IAC produces impulse responses for investment which are more in line with the empirical evidence for monetary policy shocks. Specifically, investment displays a "hump-shaped" pattern where the peak response occurs several quarters after the shock which motivates using IAC formulation (Christiano, Eichenbaum and Evans (2005)). In contrast, both no adjustment costs and capital adjustment costs lead to peak responses of investment on impact which is inconsistent with the empirical evidence.

3 Conclusion

We highlight that the real interest rate channel is central to the monetary transmission mechanism in contemporary NK models. A monetary contraction (expansion) is followed by an increase (decrease) in the real interest rate, affecting both short- and long-run real rates in the same direction. The presence of investment adjustment costs make the real interest rate channel a structural feature in this class of models.

Bibliography

Adjemian, S., Bastani, H., Juillard, M., Karamé, F., Maih, J., Mihoubi, F., Perendia, G., Pfeifer, J., Ratto, M. and Villemot, S.: 2011, Dynare: Reference Manual Version 4, *Dynare Working Papers 1*, CEPREMAP.

Basu, S. and Kimball, M. S.: 2005, Investment Planning Costs and the Effects of Fiscal and Monetary Policy. Mimeo.

Brave, S. A., Campbell, J. R., Fisher, J. D. M. and Justiniano, A.: 2012, The Chicago Fed DSGE Model, *Working Paper 02*, Federal Reserve Bank of Chicago.

Christiano, L. J., Eichenbaum, M. and Evans, C. L.: 2005, Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy, *Journal of Political Economy* **113**(1), 1–45.

Christiano, L. J., Motto, R. and Rostagno, M.: 2014, Risk shocks, *American Economic Review* **104**(1), 27–65.

Del Negro, M., Eusepi, S., Giannoni, M., Sbordone, A., Tambalotti, A., Cocci, M., Hasegawa, R. and Linder, M. H.: 2013, The FRBNY DSGE Model, *Technical Report 647*, Federal Reserve Bank of New York Staff Reports.

Galí, J.: 2015, *Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework and Its Applications*, Princeton University Press.

Groth, C. and Khan, H.: 2010, Investment adjustment costs: An empirical assessment, *Journal of Money, Credit and Banking* **42**(8), 1469–1494.

Kimball, M. S.: 1995, The Quantitative Analytics of the Basic Neomonetarist Model, *Journal of Money, Credit and Banking* **27**(4), 1241–1277.

L'Huillier, J.-P. and Yoo, D.: 2019, Where is the GE? Consumption Dynamics in DSGEs, *Journal of Money, Credit and Banking* **51**(6), 1491–1502.

Rupert, P. and Šustek, R.: 2019, On the mechanics of new Keynesian models, *Journal of Monetary Economics* **102**, 53–69.

Smets, F. and Wouters, R.: 2007, Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach, *American Economic Review* **97**(3), 586–606.

Woodford, M.: 2003, *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton University Press.

A The model

A.1 Households

The household problem is given by,

$$\text{Max} \sum_{t=0}^{\infty} \beta^t \left\{ \log\ (C_t - \varepsilon_C C_{t-1}) - \frac{L_t^{1+\eta}}{1+\eta} \right\}, \tag{A.1}$$

subject to the following budget constraint and law of motion for capital which includes investment adjustment costs (IAC) and capital adjustment costs (CAC)[9],

$$W_t L_t + R_t^k K_t + \left(\frac{1+i_{t-1}}{1+\pi_t} \right) B_{t-1} = C_t + I_t + B_t, \tag{A.2}$$

$$K_{t+1} = (1-\delta) K_t + I_t \left[1 - \frac{\Omega}{2} (\frac{I_t}{I_{t-1}} - 1)^2 \right] - \frac{\kappa}{2} (K_{t+1} - K_t)^2, \tag{A.3}$$

yielding the following equilibrium conditions,

$$\lambda_t = \frac{1}{C_t - \varepsilon_C C_{t-1}} - E_t \left\{ \frac{\beta \varepsilon_C}{C_{t+1} - \varepsilon_C C_t} \right\}, \tag{A.4}$$

$$L_t^{\eta} = \lambda_t W_t, \tag{A.5}$$

$$1 = \beta E_t \left\{ \left(\frac{\lambda_{t+1}}{\lambda_t} \right) \left(\frac{1+i_t}{1+\pi_{t+1}} \right) \right\}, \tag{A.6}$$

$$q_t (1 + \kappa (K_{t+1} - K_t)) = \beta E_t \left\{ \left(\frac{\lambda_{t+1}}{\lambda_t} \right) \left(R_{t+1}^k + (1 - \delta + \kappa (K_{t+2} - K_{t+1})) q_{t+1} \right) \right\}, \tag{A.7}$$

$$1 = q_t \left[1 - \frac{\Omega}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 - \Omega \left(\frac{I_t}{I_{t-1}} - 1 \right) \left(\frac{I_t}{I_{t-1}} \right) \right] + \beta \Omega E_t \left\{ q_{t+1} \left(\frac{\lambda_{t+1}}{\lambda_t} \right) \left(\frac{I_{t+1}}{I_t} - 1 \right) \left(\frac{I_{t+1}}{I_t} \right)^2 \right\}, \tag{A.8}$$

where λ is the Lagrange multiplier on the budget constraint and q is the ratio of the Lagrange multipliers on the law of motion for capital and the budget constraint.

A.2 Intermediate firms

Intermediate firms use a constant returns to scale technology and minimize costs subject to meeting demand. Wages and rental rates are common to all firms,

[9] We consider IAC and CAC in isolation by setting Ω or κ equal to 0.

$$\text{Min } R_t^k K_t + W_t L_t \qquad \text{s.t.} \tag{A.9}$$

$$K_t(i)^{\alpha} L_t(i)^{1-\alpha} \geq \left(\frac{P_t(i)}{P_t}\right)^{\frac{1}{\varepsilon-1}} Y_t, \tag{A.10}$$

which yields the optimal mix of capital and labour in production and marginal cost,

$$\frac{W_t}{R_t^k} = \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{K_t}{L_t}\right), \tag{A.11}$$

$$\chi_t = \left(\frac{R_t^k}{\alpha}\right)^{\alpha}\left(\frac{W_t}{1-\alpha}\right)^{1-\alpha}. \tag{A.12}$$

Letting θ denote the probability that a firm cannot adjust its prices, the firm chooses $P_t(j)$ taking into account it may not be able to change its price for a very long time and maximizes real profit,

$$\text{Max } E_t \sum_{s=0}^{\infty} (\beta\theta)^s \left(\frac{U'(C_{t+s})}{U'(C_t)}\right)\left(\frac{P_t(j)}{P_{t+s}}\left(\frac{P_t(j)}{P_{t+s}}\right)^{\frac{1}{\varepsilon-1}} Y_{t+s} - \frac{\chi_{t+s}}{P_{t+s}}\left(\frac{P_t(j)}{P_{t+s}}\right)^{\frac{1}{\varepsilon-1}} Y_{t+s}\right), \tag{A.13}$$

which yields the standard New-Keynesian Phillips Curve,

$$\pi_t = \beta E_t \pi_{t+1} + \Psi \hat{\chi}_t \tag{A.14}$$

where $\Psi = \frac{(1-\beta\theta)(1-\theta)}{\theta} \frac{1-\alpha}{1-\alpha+\frac{\alpha}{1-\varepsilon}}$.

A.3 Final goods firms

The final goods sector is perfectly competitive and aggregates intermediate inputs to produce final goods. The final goods problem is given by,

$$\text{Max } P_t Y_t - \int_0^1 P_t(i) Y_t(i), \tag{A.15}$$

where $Y_t = \left(\int_0^1 Y_t(i)^{\varepsilon}\right)^{\frac{1}{\varepsilon}}$, which yields the standard downward sloping demand function for intermediate firm i's input,

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{\frac{1}{\varepsilon-1}} Y_t, \tag{A.16}$$

that states that the demand for input i is a function of its relative price and price elasticity of demand.

A.4 Monetary policy rule and market clearing conditions

Following Rupert and Šustek (2019), we set the weight on the output gap to 0 in the Taylor rule,

$$i_t = i + \nu\pi_t + \xi_t, \tag{A.17}$$

where ξ_t is an AR(1) process with varying degrees of persistence given by,

$$\log \xi_t = \rho_m \log \xi_{t-1} + \epsilon_t, \qquad \rho_m \in (0,1), \epsilon \sim N(0, \sigma_m^2). \tag{A.18}$$

Lastly, the aggregate resource constraint states that all output is either invested or consumed,

$$Y_t = C_t + I_t \tag{A.19}$$

B Solving for the steady state

To solve for the steady state, we starting by normalizing steady state output to 1. The firm first order condition for capital then yields,

$$R^k = \frac{\alpha}{K}, \tag{B.1}$$

combining this with the steady state Euler equation for capital, we can solve for steady state capital as,

$$K = \left(\frac{\alpha}{\frac{1}{\beta} - 1 + \delta}\right). \tag{B.2}$$

From here it is straightforward to solve for steady levels of labour, consumption and investment.

C Log-linearizing the model

Log-linearizing equations (1)-(11) and the capital law of motion yields the following system of equations[10] ,

$$(1-\varepsilon_C)(1-\beta\varepsilon_C)\hat{\lambda}_t = \theta\hat{C}_{t-1} - (1+\beta\varepsilon_C^2)\hat{C}_t + \beta\varepsilon_C E_t\hat{C}_{t+1} \quad (C.1)$$

$$\hat{\lambda}_t + \hat{W}_t = \frac{\eta}{1-\alpha}\hat{Y}_t - \frac{\alpha\eta}{1-\alpha}\hat{K}_t \quad (C.2)$$

$$\hat{\lambda}_t = E_t\hat{\lambda}_{t+1} + \hat{i}_t - E_t\pi_{t+1} \quad (C.3)$$

$$\hat{L}_t = \frac{1}{1-\alpha}\hat{Y}_t - \frac{\alpha}{1-\alpha}\hat{K}_t \quad (C.4)$$

$$\hat{R}_t^k = R^k(\hat{L}_t - \hat{K}_t + \hat{W}_t) \quad (C.5)$$

$$\hat{\chi}_t = \hat{W}_t + \frac{\alpha}{1-\alpha}\hat{Y}_t - \frac{\alpha}{1-\alpha}\hat{K}_t \quad (C.6)$$

$$\pi_t = \Psi\hat{\chi}_t + \beta E_t\pi_{t+1} \quad (C.7)$$

$$\hat{i}_t^n = \upsilon\pi_t + \xi_t \quad (C.8)$$

$$\hat{Y}_t = \frac{C}{Y}\hat{C}_t + \frac{I}{Y}\hat{I}_t \quad (C.9)$$

$$\delta\hat{I}_t = \hat{K}_{t+1} - (1-\delta)\hat{K}_t \quad (C.10)$$

$$\hat{q}_t = \Omega(1+\beta)\hat{I}_t - \Omega\hat{I}_{t-1} - \beta\Omega\hat{I}_{t+1} \quad (C.11)$$

$$\hat{q}_t + \kappa K(\hat{K}_{t+1} - \hat{K}_t) = E_t\hat{\lambda}_{t+1} - \hat{\lambda}_t + E_t\hat{R}_{t+1}^k + (1-\delta)E_t\hat{q}_{t+1} + \kappa K(\hat{K}_{t+2} - \hat{K}_{t+1}) \quad (C.12)$$

D Dynare codes

All results in the paper can be recreated using the following Dynare code (parameters for persistence of shock and adjustment costs will need to be changed to match each figures specification). Alternatively, it is possible to download files to recreate the exact figures in the paper from www.joshuabrault.com/research.[11]

[10] Recall that $\hat{x}_t = \frac{x_t - x}{x}$ for all variables excluding the nominal interest rate and return on capital, which are expressed in percentage point deviations (i.e., $x_t = x_t - x$).

[11] All computations were done using Matlab 2018b and Dynare 4.5.7 (Adjemian et al. (2011)). In Dynare, capital is a predetermined variable which implies it must show up as dated $t-1$. As is customary, we lead capital by 1 period.

```
// Dynare codes for Brault and Khan (2019)
var Y C L W RK K I LAMBDA Q MC MT PI i R LRR;
varexo eps_m;
parameters  BETA ETA DELTA ALPHA THETA EPS NU PSI RHOM HABIT KAPPA OMEGA;

BETA = 0.99;
ETA = 1;
DELTA = 0.025;
ALPHA = 0.3;
NU = 1.5;
THETA = 0.7;
EPS = 0.83;
PSI = ((1-BETA*THETA)*(1-THETA)/THETA)*((1-ALPHA)/(1-ALPHA+(ALPHA/(1-EPS))));
RHOM = 0.95;
HABIT = 0;
KAPPA = 0;
OMEGA = 5.48;

model(linear);
// Some local variables
#YBAR = 1;
#KBAR = (ALPHA/((1/BETA)-1+DELTA));
#IBAR = DELTA*KBAR;
#CBAR = YBAR-IBAR;
#RKBAR = (1/BETA)-1+DELTA;

// Model equations, see Appendix C
(1-HABIT)*(1-BETA*HABIT)*LAMBDA = HABIT*C(-1) - (1+BETA*HABIT)*C + BETA*HABIT*C(+1);
LAMBDA + W = (ETA/(1-ALPHA))*Y - ((ALPHA*ETA)/(1-ALPHA))*K(-1);
LAMBDA = LAMBDA(+1) + i - PI(+1);
L = (1/(1-ALPHA))*Y - (ALPHA/(1-ALPHA))*K(-1);
RK = RKBAR*(W-K(-1)+L);
MC = W + ((ALPHA)/(1-ALPHA))*Y - ((ALPHA)/(1-ALPHA))*K(-1);
PI = BETA*PI(+1) + PSI*MC;
i = NU*PI + MT;
Y = (CBAR/YBAR)*C + (IBAR/YBAR)*I;
K = (1-DELTA)*K(-1) + DELTA*I;
MT = RHOM*MT(-1) + eps_m;
Q = OMEGA*(1+BETA)*I - OMEGA*I(-1) - BETA*OMEGA*I(+1);
Q + KAPPA*KBAR*(K-K(-1)) = LAMBDA(+1) - LAMBDA + RK(+1) + (1-DELTA)*Q(+1) + KAPPA*KBAR*(K(+1)-K);

// Define real interest rate
R = i - PI(+1);
// Long run real interest rate
-C = LRR;
end;

steady;
shocks;
var eps_m; stderr 0.01;
end;
stoch_simul(order=1, irf=40, nomoments, nograph) K Y C PI MC i R I MT LRR RK;
```