Taylor Rules and the Prospect of Indeterminacy: A Bayesian Econometric Investigation*

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Abstract

Using a Bayesian estimation method allowing for indeterminacy, we show that a policy rule responding to the output gap is a key factor, combined with a high level of trend inflation, that led to indeterminacy during the pre-Volcker years. Our evidence also suggests that a policy rule targeting output growth can potentially achieve determinacy with high trend inflation. Estimation results for the Great Moderation confirm the stabilizing powers of a policy rule targeting output growth. A more aggressive stance against inflation, combined with a stronger policy response to output growth, helped lowering trend inflation and restore determinacy during the Great Moderation. Our findings hence suggest that the Fed made a switch from a policy rule targeting output gap to one targeting output growth from the pre-Volcker years to the Great Moderation. Assuming a single policy rule to address important macroeconomic questions such as the causes of indeterminacy and the sources of the Great Moderation may be misleading.

JEL classification:

Keywords: Bayesian estimation; Indeterminacy; Taylor rules; Trend inflation; Output gap; Output growth.

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1 Introduction

There is a consensus that during the late 1960s and 1970s the US economy was in a state of indeterminacy which presumably resulted from self-fulfilling expectations and contributed to macroeconomic instability. It is only when Paul Volcker became chairman of the Federal Reserve that the economy did return to determinacy and greater stability more generally.

However, from a theoretical standpoint the exact source of indeterminacy is less agreed upon. One explanation is that the Federal Reserve was passive, failing to respond sufficiently strongly to inflation. For instance, Clarida et al. (2000) estimate policy reaction functions prior to and after the appointment of Paul Volcker and find estimates of the Taylor rule inflation response consistently lower than one for the pre-Volcker period which lead them to conclude that the economy was in a state of indeterminacy. Lubik and Schorfheide (2004) jointly estimate the parameters of a reduced form New Keynesian model using a Bayesian estimation method that explicitly accounts for the possibility of indeterminacy and reach a similar conclusion.

Yet, another explanation put forth by Coibion and Gorodnichenko (2011) (hereafter CG) is that trend inflation was high during the pre-Volcker years, and even if the Federal Reserve satisfied the Taylor principle, this would not have guaranteed determinacy.[1] It is only when trend inflation dropped to a lower level that the economy returned to determinacy. This view is shared by Hirose et al. (2020), who estimate a New-Keynesian model from which they provide evidence of indeterminacy prior to 1980 and determinacy after the early 1980s.

The present paper challenges the narrative that the Federal Reserve held to a single policy rule throughout the postwar period, while being able to restore macroeconomic stability by adopting a much more aggressive stance against inflation that possibly brought trend inflation down. Whereas the policy response to inflation and the level of trend inflation are two significant factors for the prospect of determinacy, we intend to show that the measure of economic activity targeted by the monetary authority also plays a key role in determining the stabilizing effectiveness of Taylor rules.

[1]See also Kiley (2007), Ascari and Ropele (2009), and Wolman (2011).

We provide evidence based on a New Keynesian model with positive trend inflation estimated over the pre-Volcker years and the Great Moderation using a Bayesian econometric procedure that allows explicitly for the possibility of indeterminacy. Relative to the literature investigating this question, we make two key distinctions.

First, we investigate the role of alternative Taylor rule specifications in determining how effective monetary policy is and the prospect of determinacy. We refer to these rules as *gap*, *mixed*, and *growth* policy rules, respectively. The first two rules have been widely used in the business cycle and indeterminacy literature. All assume the Fed smooths short-run movements in nominal interest rates. The gap rule also states the Fed responds to deviations of inflation from a fixed target and current output gap.[2] The mixed rule states the Fed responds to deviations of inflation from a fixed target, current output gap, and deviations of current output growth from trend growth. Finally, the growth rule states that the Fed responds to deviations of inflation from a fixed target and deviations of current output growth from trend growth.

Why consider these different Taylor rule specifications? For one, Walsh (2003) has argued that despite the wide acceptance of policy rules responding both to inflation and the output gap, "it is not clear that inflation and output gap stabilization are the objectives central banks either should or actually do pursue in the conduct of policy". He points that in 2000 the Fed explicitly stated that it was targeting the growth in output relative to the growth in potential, rather than the output gap itself.

Another reason in CG (2011) based on a calibrated New Keynesian model with positive trend inflation is that when trend inflation exceeds 5%, a rule targeting the output gap will achieve determinacy only if the policy response to inflation deviates significantly from the Taylor principle compared to a rule targeting output growth. Khan et al. (2020) further demonstrate that a rule targeting the output gap or output gap and output growth represents a threat to determinacy for a policy response to the output gap of about 0.15 or higher, and this even at low rates of trend inflation like 2% and 3%. Moreover, they find that with a policy rule targeting the output gap and output growth, the influence of output gap on

[2]The output gap is defined as the difference between short-run output and output at perfectly flexible prices.

the determinacy outcome is disproportionate relative to that of output growth. Assuming instead that the Fed targets output growth, they show determinacy can be achieved with a policy response to inflation at or near the Taylor principle up to a level of trend inflation of 7%.[3]

With the help of a Bayesian estimation procedure permitting the possibility of indeterminacy, Haque et al. (2021) report some evidence of determinacy during the pre-Volcker years. They focus on a single policy rule responding to inflation, the output gap and output growth. They amend the standard New Keynesian price-setting model in several ways. Adding nominal wage rigidity to the standard model results in a 100% probability of indeterminacy. The one model avoiding indeterminacy embeds consumption composed of a basket of imported oil and domestically produced goods, oil serving as an input, and a real wage rigidity resulting from modifying the intratemporal optimality condition derived from the household's problem which Blanchard and Galí (2007) qualify of "admittedly ad hoc". By contrast, we emphasize the key role played by alternative Taylor rules for the determinacy outcome rather than amendments made to the standard model under a single policy rule.

The second key distinction we make is to investigate the determinacy properties over a plausible range of priors for the Calvo price stickiness parameter. We choose priors that are broadly consistent with micro price adjustment evidence (Bils and Klenow (2004); Nakamura and Steinsson (2008)). We intend to show that when monetary policy targets the output gap or the output gap and output growth, the estimated frequency of price adjustment is very sensitive to modest variations in the prior of the price stickiness parameter. We show this significantly affects the determinacy outcome, especially during the Great Moderation. By contrast, when the Fed targets output growth, we find that estimates of the price stickiness parameter exhibit greater stability upon varying the prior of the price stickiness parameter.

We use a New Keynesian model with positive trend inflation. The representative consumer maximizes the present discounted stream of utility over consumption and firm-specific labor. A real adjustment friction takes the form of consumer habit formation. We focus on

[3]That is, determinacy is obtained for a policy response to inflation complying with the Taylor principle or quite close to it.

models free of ad hoc backward-looking price-setting elements such as rule-of-thumb behavior or the indexation to past inflation and/or steady state inflation for reasons which are explicitly stated in Section 2. We estimate the model over two subsamples of US data that are 1966:Q1-1979:Q2 and 1982:Q4-2008:Q4. They represent periods of "high" and "low" trend inflation, respectively. We explore how alternative policy rules interact with different levels of trend inflation to determine whether the economy is in a state of determinacy or not.

A first set of findings pertains to the pre-Volcker period. Based on point estimates of either the gap, mixed or growth policy rule, we find evidence of active monetary policy during the pre-Volcker period. That is, all rules indicate monetary policy was complying with the Taylor principle requiring a policy response to inflation greater than one. Interestingly, based on 90% confidence intervals, we report several cases for which there is no uncertainty that the Fed's response to inflation was active during the pre-Volcker period. We find policy inflation responses close to Taylor (1993)'s original prescription of 1.5 and even exceeding it. Our evidence seems consistent with Orphanides (2002)' claim that monetary policymakers satisfied the Taylor principle even before the entry of Paul Volcker as chairman of the Federal Reserve.

We show that the policy rule posing the most serious threat to determinacy is one targeting the output gap. We find that the probability of indeterminacy in the pre-Volcker period is always close to 1. It is followed by the mixed rule implying indeterminacy with certainty (probability of determinacy less than 1%) or near-certainty (probability of determinacy less than 5%). A factor contributing to indeterminacy under the gap and mixed policy rules is that our estimates systematically indicate the policy response to the output gap was significantly stronger during the pre-Volcker period than after the early 1980s, and this by a factor of 2 to 4. Interestingly, we find estimates of trend inflation consistent with the average annualized rates of inflation observed during the two subperiods. We think this is an interesting consequence of the estimation method which allows explicitly the possibility of indeterminacy. A Bayesian method precluding this possibility has problems capturing movements in trend inflation between subperiods as noted by Smets and Wouters (2007).

Our results are very different for the pre-Volcker period when the policy rule targets only output growth. For then, we find that despite high trend inflation the probability of determinacy can reach 95%. It is worth noticing that we do not have to resort to indexation to obtain this finding. The growth rule represents much less threat to determinacy than the other two types of rules because it precludes the use of the output gap which has a detrimental impact on the determinacy outcome.

When considering the Great Moderation, we find that when the Fed targets only the output gap, we get a determinacy result only if the prior on the price stickiness parameter is low. For somewhat higher but still plausible values of this prior, our estimates suggest the economy is in state of indeterminacy with certainty. We find somewhat similar results for the mixed policy rule.

The factors explaining these results are the following. With either the gap or the mixed policy rule, we find that the estimated degree of price stickiness rises very significantly with relatively modest increases in the prior of the price stickiness parameter. Taking the mixed rule for example, we find that the Calvo probability of price stickiness rises from 0.48 to 0.76 if the prior increases only from 0.55 to 0.65. It is the combination of an estimated trend inflation at nearly 3% and an estimated higher degree of price rigidity that poses a threat to determinacy under the gap and the mixed rule during the Great Moderation.

By comparison, the output growth policy rule achieves determinacy with certainty during the Great Moderation. A first reason why the output growth rule poses no threat to determinacy is that the estimated trend inflation is low. A second reason is that the growth rule not only precludes a reaction to the output gap, but also implies that the estimated policy response to output growth was much stronger during the Great Moderation than during the pre-Volcker years. Finally, the estimated price stickiness parameter is lower and more stable upon varying the prior of the price stickiness parameter under the growth rule than under either the gap or the mixed rule, which also favors a determinacy outcome.

We conclude from our empirical investigation that either the Federal Reserve never followed a policy rule that resulted in a state of indeterminacy during the pre-Volcker years and the Great Moderation, or that it did follow a rule minimally targeting the output gap that resulted in indeterminacy during the pre-Volcker years, but then switched to a rule targeting output growth only which helped achieving determinacy during the Great Moderation. Therefore, assuming a single policy rule to address such important macroeconomic questions as the causes of indeterminacy and the sources of the Great Moderation may be misleading.

The rest of the paper is organized as follows. Section 2 presents our New-Keynesian model with trend inflation and the three monetary policy rules that we consider. Section 3 discusses the data and our estimation strategy. Section 4 analyzes our estimation results and implications for determinacy for the Great Inflation and Section 5 for the Great Moderation. Finally, Section 6 concludes.

2 The Model

Our model is a New-Keynesian framework with firm-specific labor and positive trend inflation. We abstract from capital accummulation. There is consumer habit formation. Price stickiness is determined by a Calvo (1983) probability of price non-reoptimization firms face in each period. Aggregate fluctuations are driven by shocks to the discount rate, TFP, and monetary policy, and if in a state of indeterminacy, sunspot shocks.

2.1 Backward-Looking Price Setting Devices

The omission of backward-looking price-setting devices deserves explanation. Galí and Gertler (1999) assume rule-of-thumb behavior on the part of some price setters. That is, upon receiving a Calvo-signal allowing them to change their prices, a fraction of firms will do so in an optimal forward-looking way while the remaining fraction of firms will simply index their prices to the previous period rate of inflation. Firms not receiving the signal to change their prices will stay put.

This amendment to the standard New Keynesian price-setting model justifies the presence of the previous period rate of inflation into the New Keynesian Phillips Curve. While Galí and Gertler show the fraction of rule-of-thumb price-setting firms need not be large to improve the fit of the data, there remains nonetheless the question of what explains in reality that when allowed to change their prices, a fraction of firms will understand optimal price-setting while the remaining fraction will not not. Furthermore, it is hard to see why rule-of-thumbers would not learn rapidly from optimal price-setters, or why they would not be able to overcome some informational or organizational problems that would prevent them from setting prices optimally.

Another price-setting device introduced by Christiano et al. (2005) is to assume that all firms receiving the Calvo-signal of a price change will do so in an optimal forward-looking way, while the remaining fraction of firms will index their prices to the previous quarter rate of inflation rather than stay put.[4]

Now, both of these amendments to the purely forward-looking New Keynesian model do not rely on microeconomic foundations. Furthermore, in the case of indexation, this assumption unlike rule-of-thumb price-setting behavior counterfactually implies that all prices in the economy change at a quarterly pace, something which is simply inconsistent with evidence from micro-level consumer price data.

Backward-looking price-setting devices, and indexation in particular, have been questioned by several macroeconomists. For instance, regarding price indexation, Woodford (2007, p. 204) states that "the model's implication that prices should continuously adjust to changes in prices elsewhere in the economy flies in the face of the survey evidence." Cogley and Sbordone (2008, p. 2101) mention that backward-looking price indexation lacks "a convincing microeconomic foundation." Chari et al. (2009, p. 261) argue that "this feature is inconsistent with microeconomic evidence on price setting." (Christiano 2015, p. 354) holds that the "no-indexation assumption is suggested by the same microeconomic observations that motivate price setting frictions in the first place."

Furthermore, Ascari et al. (2018) emphasize that price indexation also has the undesirable property that it delivers inertial inflation responses not only to monetary policy shocks, but to non-monetary shocks like TFP shocks as well, a finding which is generally at odds with available VAR evidence. Phaneuf et al. (2018) show that the indexation assumption drives several of the main findings obtained within this class of models like the persistent

[4] A variant of this assumption holds instead that firms not allowed to change their prices can index them partly to past inflation and partly to the steady state rate of inflation.

and hump-shaped response of inflation to a monetary policy shock reported by Christiano et al. (2005).

2.2 Representative Consumer

The representative consumer maximizes expected utility over final consumption goods C and differentiated labour L

$$\underset{C_t, L_t(i), B_{t+1}}{\text{Max}} \quad E_0 \sum_{t=0}^{\infty} \beta^t b_t \left[\ln(C_t - h\bar{C}_{t-1}) - \frac{1}{1+\eta^{-1}} \int_0^1 L_t(i)^{1+\eta^{-1}} di \right], \tag{1}$$

where β is the subjective discount factor, h the degree of external habit formation, and η the elasticity of labour supply. The representative consumer is subject to the following budget constraint

$$P_t C_t + B_{t+1} = \int_0^1 W_t(i) L_t(i) di + B_t R_t + T_t, \tag{2}$$

where B_t is the stock of nominal bonds that the household enters the period with, $W_t(i)$ is the nominal wage page by sector i, P_t is the price of the final consumption good, R_t is the (gross) nominal interest rate, and T_t is profits from ownership of the firms. b_t is an intertemporal preference shock which follows an AR(1) process given by

$$\ln b_t = (1 - \rho_b) \ln b + \rho_b \ln b_{t-1} + \epsilon_t^b. \tag{3}$$

The necessary first order conditions for the representative consumer are given by

$$1 = \beta E_t \left[\frac{b_{t+1}}{b_t} \left(\frac{C_t - h\bar{C}_{t-1}}{C_{t+1} - h\bar{C}_t} \right) \left(\frac{R_t}{\pi_{t+1}} \right) \right], \tag{4}$$

$$L_t(i)^{\eta^{-1}} = \frac{w_t(i)}{C_t - h\bar{C}_{t-1}}, \tag{5}$$

where $\pi_t \equiv P_t / P_{t-1}$ and real wage is defined as $w_t = W_t / P_t$.

2.3 Final Goods Firms

Final goods firms combine intermediate inputs into a single aggregate output good using a CES aggregator. The model features no government consumption or capital accumulation so that aggregate output is equal to aggregate consumption.

$$C_t = Y_t = \left[\int_0^1 Y_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}. \tag{6}$$

θ is the elasticity of substitution between intermediate inputs. Final goods firms maximize profits subject to intermediate input costs which is given by

$$\underset{Y(i)}{\text{Max}} \;\; P_t Y_t - \int_0^1 P_t(i) Y_t(i) di. \tag{7}$$

The first order condition for final goods firms yields the standard downward sloping demand curve for each intermediate good i

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\theta} Y_t, \tag{8}$$

and the aggregate price index given by

$$P_t = \left(\int_0^1 P_t(i)^{1-\theta} di \right)^{\frac{1}{1-\theta}}. \tag{9}$$

2.4 Intermediate Goods Firms

Intermediate goods are produced by a continuum of monopolistically competitive firms with a constant returns to scale production function given by

$$Y_t(i) = A_t L_t(i), \tag{10}$$

where A_t is the aggregate productivity level governed by an AR(1) process given by

$$\ln A_t = \ln \bar{A} + \ln A_{t-1} + \epsilon_t^A. \tag{11}$$

Intermediate goods firms minimize costs subject to meeting demand. The cost minimization problem for firm i is given by

$$\underset{L_t(i)}{\text{Min}} \;\; W_t L_t(i) \tag{12}$$

subject to

$$A_t L_t(i) \geq \left(\frac{P_t(i)}{P_t}\right)^{-\theta} Y_t. \tag{13}$$

The first-order condition for the cost minimization problem yields firm i's nominal marginal cost

$$MC_t(i) = \frac{W_t(i)}{A_t}. \tag{14}$$

Firms are subject to sticky prices via the Calvo mechanism. Each period firms have a probability of being able to change their price given by $1 - \xi_p$. A firm setting its price optimally in period t is cognizant of the fact that they may not be able to reset their price a long time and maximizes the following flow of profits given by

$$\text{Max} \;\; E_t \sum_{\tau=0}^{\infty} \xi_p^\tau \Lambda_{t,t+\tau} Y_{t+\tau}(i) \left[P_t(i) - MC_{t+\tau}(i)\right], \tag{15}$$

where $\Lambda_{t,t+\tau} \equiv \beta^\tau \frac{\Lambda_{t+\tau}}{\Lambda_t}$ and Λ_t is the marginal utility of nominal income to the representative consumer in period t. The first-order condition that determines the optimal price yields

$$\frac{P_t^\star(i)}{P_t} = \left(\frac{\theta}{\theta - 1}\right) E_t \frac{\sum_{\tau=0}^{\infty} \xi_p^\tau \Lambda_{t,t+\tau} \left(\frac{P_{t+\tau}}{P_t}\right)^{1+\theta} Y_{t+\tau} mc_{t+\tau}(i)}{\sum_{\tau=0}^{\infty} \xi_p^\tau \Lambda_{t,t+\tau} \left(\frac{P_{t+\tau}}{P_t}\right)^{\theta} Y_{t+\tau}}, \tag{16}$$

where $mc_t(i)$ is the firm-specific real marginal cost (i.e., $mc_t(i) \equiv \frac{MC_t(i)}{P_t}$) and $P_t^\star(i)$ is the optimal reset price. Since all firms which can reset their price choose the same price, we drop the firm-specific subscript in the following derivations.

Using equations (5), (8), (10), and (14), it is straightforward to show that the optimal price reset equation is given by

$$\left(\frac{P_t^\star}{P_t}\right)^{1+\frac{\theta}{\eta}} = \left(\frac{\theta}{\theta-1}\right) E_t \frac{\sum_{\tau=0}^{\infty} \xi_p^\tau \Lambda_{t,t+\tau} \left(\frac{P_{t+\tau}}{P_t}\right)^{1+\theta(1+\eta^{-1})} \left(\frac{Y_{t+\tau}}{A_{t+\tau}}\right)^{1+\eta^{-1}} (Y_{t+\tau} - hY_{t+\tau-1})}{\sum_{\tau=0}^{\infty} \xi_p^\tau \Lambda_{t,t+\tau} \left(\frac{P_{t+\tau}}{P_t}\right)^{\theta} Y_{t+\tau}}, \quad (17)$$

and the associated output that would prevail under flexible prices is given by

$$\left(\frac{Y_t^F}{A_t}\right)^{1+\eta^{-1}} = \frac{\theta-1}{\theta} + h\left(\frac{Y_t^F}{A_t}\right)^{\eta^{-1}} \left(\frac{Y_{t-1}^F}{A_t}\right). \quad (18)$$

2.5 Monetary Policy

We consider three different Taylor type rules. Each rule is characterized by interest rate smoothing. The first rule, which we refer to as the *mixed* policy rule, states that the Fed responds to deviations of current inflation from a fixed target, the current output gap, and deviations of current output growth from trend growth

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_R} \left[\left(\frac{\pi_t}{\pi}\right)^{\alpha_\pi} \left(\frac{Y_t}{Y_t^F}\right)^{\alpha_y} \left(\frac{Y_t}{Y_{t-1}} g_Y^{-1}\right)^{\alpha_{dy}}\right]^{1-\rho_R} \epsilon_t^R. \quad (19)$$

The second rule, which we refer to as the *gap* policy rule, states that the Fed responds to deviations of current inflation from a fixed target and the current output gap

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_R} \left[\left(\frac{\pi_t}{\pi}\right)^{\alpha_\pi} \left(\frac{Y_t}{Y_t^F}\right)^{\alpha_y}\right]^{1-\rho_R} \epsilon_t^R. \quad (20)$$

The final rule, which we refer to as the *growth* policy rule, states that the Fed responds to deviations of current inflation from a fixed target and current output growth from trend growth

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_R} \left[\left(\frac{\pi_t}{\pi}\right)^{\alpha_\pi} \left(\frac{Y_t}{Y_{t-1}} g_Y^{-1}\right)^{\alpha_{dy}} \right]^{1-\rho_R} \epsilon_t^R. \tag{21}$$

2.6 Log-Linearization

Economic growth stems from neutral technological progress. Output growth is denoted g_Y. Solving the model requires detrending output, which is done by removing trend growth and taking a log-linear approximation of the stationary model around the non-stochastic steady state.

3 Model Solution, Estimation, and Data

3.1 Rational expectations solution under indeterminacy

To solve the model when there are potentially many equilibrium we follow the solution method proposed by Lubik and Schorfheide (2003). The linear rational expectations (LRE) model can be represented in its canonical form which is given by

$$\boldsymbol{\Gamma}_0(\boldsymbol{\vartheta})\mathbf{s}_t = \boldsymbol{\Gamma}_1(\boldsymbol{\vartheta})\mathbf{s}_{t-1} + \boldsymbol{\Psi}(\boldsymbol{\vartheta})\boldsymbol{\epsilon}_t + \boldsymbol{\Pi}(\boldsymbol{\vartheta})\boldsymbol{\eta}_t, \tag{22}$$

where $\mathbf{s}_t$ is a vector of endogenous variables, $\boldsymbol{\epsilon}_t$ is a vector of exogenous structural disturbances, and $\boldsymbol{\eta}_t$ is a vector of one-step ahead forecast errors for the expectational variables in the model. $\boldsymbol{\Gamma}_0(\boldsymbol{\vartheta})$, $\boldsymbol{\Gamma}_1(\boldsymbol{\vartheta})$, $\boldsymbol{\Psi}(\boldsymbol{\vartheta})$, and $\boldsymbol{\Pi}(\boldsymbol{\vartheta})$ are matrices containing potentially non-linear combinations of the structural parameters. Following Lubik and Schorfheide (2003), the full set of LRE solutions can then be characterized by,

$$\mathbf{s}_t = \boldsymbol{\Phi}(\boldsymbol{\vartheta})\mathbf{s}_{t-1} + \boldsymbol{\Phi}_{\boldsymbol{\epsilon}}(\boldsymbol{\vartheta}, \tilde{\boldsymbol{M}}) + \boldsymbol{\Phi}_{\boldsymbol{\zeta}}(\boldsymbol{\vartheta}, \boldsymbol{M}_{\boldsymbol{\zeta}})\boldsymbol{\zeta}_t, \tag{23}$$

where $\boldsymbol{\Phi}(\boldsymbol{\vartheta})$, $\boldsymbol{\Phi}_{\boldsymbol{\epsilon}}(\boldsymbol{\vartheta}, \tilde{\boldsymbol{M}})$, and $\boldsymbol{\Phi}_{\boldsymbol{\zeta}}(\boldsymbol{\vartheta}, \boldsymbol{M}_{\boldsymbol{\zeta}})$ are matrices of the coefficients. Following Lubik and Schorfheide (2004) we impose that $\boldsymbol{M}_{\boldsymbol{\zeta}} = 1$, implying that $\boldsymbol{\zeta}_t$ is no longer a vector of sunspot shocks, but instead a reduced form sunspot shock (now given by ζ_t) which captures a sunspot shock to all of the expectational variables in the system. ζ_t is assumed to follow a process given by $\zeta_t \sim$ i.i.d. $N(0, \sigma_\zeta^2)$.

Under this setup, indeterminacy can impact endogenous variables in two different ways. First, the non-fundamental expectation errors , ζ_t, can impact model dynamics. Second, because the model features multiplicity of equilibrium the propagation structural shocks, $\boldsymbol{\epsilon}_t$, is not unique and is in part determined by the arbitrary matrix $\tilde{\boldsymbol{M}}$.

Consistent with the literature following Lubik and Schorfheide (2004), we replace $\tilde{\boldsymbol{M}}$ with $\boldsymbol{M}^\star(\boldsymbol{\theta}) + \boldsymbol{M}$. $\boldsymbol{M}^\star(\boldsymbol{\theta})$ is found by minimizing the difference between the impact responses of endogenous variables to structural disturbances at the boundary between determinacy and indeterminacy, where the boundary is found by perturbing the Taylor Rule response to inflation, α_π. The remaining matrix $\boldsymbol{M}$ is estimated from the data.

3.2 Econometric Strategy

We use a full information Bayesian estimation strategy to characterize the posterior distributions of the structural parameters and shocks across the determinacy and indeterminacy regions of the model. The main challenge is that the associated posterior distributions, particularly for the Taylor rule inflation response parameter, are multi-modal and difficult to estimate using standard methods (such as the Random-walk Metropolis Hastings algorithm). We follow Hirose et al. (2020) and use the sequential Monte Carlo (SMC) algorithm proposed by Herbst and Schorfheide (2016, 2014), which allows us to approximate the posteriors in both the determinacy and indeterminacy regions in a single estimation.

In this case the likelihood function is given by

$$p(\boldsymbol{X}^T|\boldsymbol{\vartheta}_S, S) = 1\{\boldsymbol{\vartheta}_S \in \boldsymbol{\Theta}^D\} p^D(\boldsymbol{X}^T|\boldsymbol{\vartheta}_D, D) + 1\{\boldsymbol{\vartheta}_S \in \boldsymbol{\Theta}^I\} p^I(\boldsymbol{X}^T|\boldsymbol{\vartheta}_I, I), \tag{24}$$

where $\boldsymbol{\Theta}^D$ and $\boldsymbol{\Theta}^I$ are the determinacy and indeterminacy regions of the parameter space, $p^D(\boldsymbol{X}^T|\boldsymbol{\vartheta}_D, D)$ and $p^I(\boldsymbol{X}^T|\boldsymbol{\vartheta}_I, I)$ the likelihood functions under determinacy and indeterminacy, and finally $1\{\boldsymbol{\vartheta}_S \in \boldsymbol{\Theta}^D\}$ and $1\{\boldsymbol{\vartheta}_S \in \boldsymbol{\Theta}^I\}$ are indicator functions which equal 1 if the parameters are in the determinacy or indeterminacy region.

The SMC algorithm constructs a sequence of tempered posteriors given by

$$\pi_n(\boldsymbol{\vartheta}) = \frac{[p(\boldsymbol{X}^T|\boldsymbol{\vartheta}_S, S)]^{\phi_n} p(\boldsymbol{\vartheta}_S|S)}{\int_{\boldsymbol{\vartheta}_S} [p(\boldsymbol{X}^T|\boldsymbol{\vartheta}_S, S)]^{\phi_n} p(\boldsymbol{\vartheta}_s|S) d\boldsymbol{\vartheta}_S}, \tag{25}$$

where ϕ_n for $n = 1, \ldots, N_\phi$ is a sequence that slowly increases from zero to one. As noted in (Herbst and Schorfheide 2016, p. 76), the algorithm consists of three primary steps: 1) correction, which reweights particles to reflect the density in iteration n; 2) selection, which eliminates particle degeneracy by resampling; and 3) mutation, which propagates the particles forward using a Markov transition kernel to adapt to the current bridge density. Our estimation uses $N = 10,000$, $N_\phi = 200$, and $\lambda = 2$.[5]

After obtaining approximations to the posterior distributions we compute the posterior probability of determinacy as

$$P(\boldsymbol{\vartheta} \in \boldsymbol{\Theta}^D|\boldsymbol{X}^T) = \frac{1}{N}\sum_{i=1}^{N} 1\{\boldsymbol{\vartheta}_{N_\phi} \in \boldsymbol{\Theta}^D\}, \tag{26}$$

effectively counting the number of draws which yield determinacy in the final stage of the SMC algorithm.

3.3 Data

To estimate a subset of the structural parameters of the model we use three U.S. quarterly time series: per capita real GDP growth, GDP deflator based inflation, and the Federal Funds rate. These data are obtained from the St. Louis Federal Reserve Economic Database (FRED). We report the reference codes for the series and the variable construction in the Appendix A.

We estimate the model's parameters over two different samples. The first sample corresponds to the Great Inflation period and runs from 1966Q1 to 1979Q2. The second sample

[5]This imposes a tempering schedule according to $\phi_n = \left(\frac{n-1}{N_\phi - 1}\right)^\lambda$.

corresponds to the Great Moderation period and runs from 1982Q4 to 2008Q4.[6]

The observables are mapped into the model in the following manner,

$$\begin{bmatrix} 100 \log \Delta Y_t \\ 100 \log \Delta P_t \\ 100 \log R_t \end{bmatrix} = \begin{bmatrix} \bar{A} \\ \bar{\pi} \\ \bar{R} \end{bmatrix} + \begin{bmatrix} \tilde{y}_t - \tilde{y}_{t-1} + \epsilon_t^A \\ \tilde{\pi}_t \\ \tilde{R}_t \end{bmatrix}, \tag{27}$$

where $\bar{A}$, $\bar{\pi}$, and $\bar{R}$ are the steady state values of the growth rate of output, inflation, and the nominal interest rate, respectively. These values are expressed in net terms given by $100(\bar{A}-1)$, $100(\bar{\pi}-1)$, and $100(\bar{R}-1)$.

3.4 Calibration and Prior Distributions

All parameters are estimated except two which are calibrated. The first is the elasticity of substitution between differentiated goods which is set equal to 9, implying a 12.5% price markup with zero trend inflation. The other is the Frisch elasticity of labor supply which is set to 1. These values are relatively standard calibrations in the literature.

The remaining structural parameters in the model are estimated. The habit formation parameter has a prior mean of 0.7 with a prior standard deviation of 0.1. We explore a range of plausible Calvo price rigidity priors given by a beta distribution with means of 0.5, 0.55, and 0.65 for the pre-Volcker years and means 0.5, 0.55, 0.65 and 0.75 for the Great Moderation. These mean priors reflect values assumed in the literature. For example, Smets and Wouters (2007) assign a mean prior 0.5 to the Calvo price stickiness parameter based on evidence in Bils and Klenow (2004). Coibion and Gorodnichenko (2011) assume the Calvo probability of price non-reoptimization is 0.55. Justiniano et al. (2010, 2011) use a mean prior of 2/3 following the evidence in Nakamura and Steinsson (2008). Finally, Justiniano and Primiceri (2008) assume a mean prior of 0.75 which we see as an upper limit. In each case the prior standard deviation is equal to 0.1.

Priors for the average rate of inflation, growth rate of output, and nominal interest rate are set to their average over the 1966Q1 to 2008Q4 sample. We use relatively diffuse priors

[6] The exclusion of the periods from late 1979 to early 1982 is standard (e.g., Clarida et al. (1999)) since during this period the Federal Reserve was not following an interest rate target.

around these means. The average rate of inflation and output growth have normal distribution priors with means of 0.985 and 0.370, and standard deviations of 0.75 and 0.15. The average nominal interest rate has gamma distribution prior with mean equal to 1.597 and standard deviation equal to 0.25.

The parameters describing monetary policy are standard. The interest rate smoothing coefficient features a prior mean of 0.6 and a standard deviation of 0.2. The policy response to inflation is characterized by a gamma distribution with prior mean equal to 1.5 and standard deviation equal to 0.3. Both the policy response to output growth and output gap have gamma priors with means of 0.125 and standard deviations of 0.1. When estimating the growth or gap rule, we calibrate the gap or growth response to zero.

The exogenous disturbances of the model have standard priors. Preference, neutral technology, monetary policy, and sunspot shocks have inverse gamma priors with a mean of 0.5 and standard deviation of 4. Additionally, for the correlation between the sunspot shock and the structural shocks we use normal distributions with mean 0 and standard deviation equal to 1. Persistence parameters for preference shocks, neutral technology, and monetary policy have priors given by a beta distribution with mean 0.5 and standard deviation equal to 0.2.

4 Estimation Results: The Pre-Volcker Period

This section presents and analyzes our results for the pre-Volcker period (1966:Q1 to 1979:Q2). We report estimates for the gap, mixed and growth policy rule models. We also provide estimates of structural parameters and shocks for mean priors assigned to the Calvo price stickiness parameter of 0.5, 0.55 and 0.65, with 90% confidence intervals for the estimated parameters. The $\log p(X^T)$ represents the marginal data density of a particular model, while *Prob(det)* is the posterior probability of equilibrium determinacy implied by that particular model.

4.1 Gap and Mixed Policy Rule Models

Table 1 presents the estimation results for the *gap-rule* model. We find evidence of indeterminacy with certainty (less than one percent probability of determinacy) for the three mean priors of the Calvo price stickiness parameter ξ_p. A factor explaining the indeterminacy result under the gap-rule is the high estimated trend inflation. We obtain estimates of average inflation implying an average annualized rate of inflation between 5.34% and 6.26% depending on the prior assigned to the price stickiness parameter. Under the gap-rule, such high levels of trend inflation represent a threat to determinacy unless the policy response to inflation largely deviates from the Taylor principle (Khan et al. 2020).

We find estimates of the policy responses to inflation which are greater than 1, but that do not deviate widely from 1. Depending on the mean prior of the price stickiness parameter, these responses can even exceed Taylor (1993)'s original prescription of 1.5, with an estimate of 1.67 with prior $\xi_p = 0.55$, for example. Based on the 90% confidence intervals for the policy response to inflation, we find no evidence that α_π can be lower than 1. In other words, monetary policy was moderately active during the pre-Volcker period under the gap-rule.

The second key factor responsible for indeterminacy under the gap-rule is the interest rate responses to the output gap. We find estimates that lie between 0.26 and 0.33 for this parameter depending on the prior assigned the price stickiness parameter. Based on calibrated New Keynesian models, Coibion and Gorodnichenko (2011) and Khan et al. (2020) provided evidence that policy responses to output gap of this size require very strong interest rate responses to inflation to achieve determinacy, of an order of magnitude of 4 to 6 for trend inflation rates of 5% to 6%. Our findings empirically confirm that targeting the output gap during the pre-Volcker years yields indeterminacy.

Table 2 presents estimation results corresponding to the *mixed-rule* model with policy responses to the output gap and output growth. Our evidence here suggests that the economy was in a state of indeterminacy with certainty (less than one percent probability of determinacy) or near-certainty (less than five percent probability of determinacy) during the pre-Volcker period depending on the prior of the price stickiness parameter. The key factors behind these indeterminacy results are essentially the same as for the gap-rule. The estimated annualized average rate of inflation is high–between 5.57% and 5.94%. Monetary policy is moderately active with estimated policy responses to inflation between 1.495 and 1.613. The policy responses to the output gap are still quite high with estimated parameters between 0.24 and 0.28. The estimated policy responses to output growth are relatively low at about 0.12.

4.2 The Growth Policy Rule Model

Table 3 presents the estimation results for the growth-rule model. We find a probability of determinacy under the growth-rule going from 95% to 44% for mean priors of the price stickiness parameter going from 0.5 to 0.65. The estimated average annualized rate of inflation ranges from 5.416% to 5.484%. Therefore, estimates of trend inflation are less sensitive to the choice of a prior of ξ_p under the growth-rule than under the gap-rule and the mixed-rule. What helps the growth-rule model achieving determinacy is that it precludes a policy response to the output gap. Furthermore, the estimated policy responses to output growth under the growth-rule are nearly 0.2, which favors determinacy. But another factor that plays a role in obtaining the determinacy result is that the estimated Calvo price stickiness parameters are systematically lower for all priors assigned to ξ_p than those estimated in either the gap-rule model or the mixed-rule model.

We have also estimated the three policy rule models for a longer sample of data for the pre-Volcker period, that is, from 1960:Q1 to 1979:Q2. We do not formally report these findings to save space. We found that the gap-rule and mixed-rule models imply indeterminacy with certainty for all priors of the Calvo probability of price stickiness. For the longer pre-Volcker sample period, the growth-rule model predicts a state of determinacy with a probability of 99% to 62.4% depending on the prior of ξ_p.[7] When looking at the marginal data density $\log P(X^T)$ predicted by alternative models, we find that the gap-rule and mixed-rule models are preferred to the growth-rule model for the pre-Volcker period.

[7]The reader interested in the detailed results is referred to an on-line appendix at the following address: https://braultjosh.github.io/pdfs/BP_Appendix.pdf.

What can we learn from these findings from the pre-Volcker period? The existing literature on the prospect of determinacy in economies with positive trend inflation has established that policy rules minimally targeting the output gap are more destabilizing than a policy rule targeting output growth. Our estimation results seem to confirm this. Both the gap-rule and the mixed-rule models seem to better describe the macroeconomic instability of the pre-Volcker years. The indeterminacy result under the gap and mixed policy rules are high estimated rates of trend inflation and the policy responses to the output gap.

At the same time, our evidence for the pre-Volcker years reveals the potentially stabilizing power of a policy rule targeting output growth. That is, our empirical findings indicate that the growth-rule model complies with the pre-Volcker period observables while potentially achieving determinacy.

5 Estimation Results: The Great Moderation

We now present and analyze our results for the Great Moderation period (1982:Q4 to 2008:Q4).

5.1 Gap and Mixed Policy Rule Models

The estimation results obtained with the gap-rule model for the Great Moderation are presented in Table 4. A first thing to note is that the estimated average annualized rates of inflation drop significantly from their pre-Volcker period levels. We find estimates between 2.82% and 3.64%. Therefore, the estimated gap-model captures the decline in trend inflation that took place after 1982. At the same time, we find significant increases in the policy response to inflation, α_π, which is now equal to or exceeds 2.31 depending on the prior for the price stickiness parameter.

When looking at the posterior probability of equilibrium determinacy implied by the gap-rule model, we find that it is quite sensitive to modest variations in the mean prior assigned to the Calvo probability of price stickiness parameter. That is, for prior 0.5, the probability of determinacy is estimated at 87%. Besides the lower level of trend inflation and the more hawkish policy stance against inflation, the other factor contributing to the determinacy result is the low estimate for the Calvo probability of price non-reoptimization at 0.433.

When priors of the price stickiness parameter are increased to either 0.55 or 0.65, we find an indeterminate state with certainty. The key factors explaining this drastic change in the determinacy outcome are the following. The first factor is that estimated trend inflation increases with the prior of ξ_p. The second is the estimated average waiting time between price adjustment which also increases with higher priors of ξ_p. For instance, it increases from once every 5.3 months on average with prior 0.5 to once every 12 months for the slightly higher prior of 0.55, and once every 12.8 months with prior 0.65. Note that this time the policy responses to the output gap are much weaker than those estimated for the pre-Volcker years. Therefore, they are not a key factor behind our indeterminacy result for the period of the Great Moderation.

Table 5 presents estimation results for the mixed-rule model. They are broadly the same as those reported for the gap-rule model, with a notable exception. This time, we find the economy is in a state of determinacy for priors of the price stickiness parameter of 0.5 and 0.55. Then with prior 0.65, we obtain an estimated 97% probability of indeterminacy. The key factors delaying obtention of the indeterminacy result until reaching prior 0.65 of ξ_p are an average waiting time between price adjustment of less than 6 months with priors 0.5 and 0.55, and policy responses to output growth of 0.45 and 0.48, respectively. When the prior of ξ_p is set at 0.65, the average frequency of price changes jumps to once every 12.2 months, and the parameter governing the policy response to output growth drops to 0.26.

5.2 The Growth Policy Rule Model

Table 6 reports the estimation results for the growth-rule model. For this particular model, we report estimates for priors of the Calvo price stickiness parameter of 0.5, 0.65 and 0.75. A significant difference between the growth-rule model and either the gap-rule or mixed-rule model is the overall stability of estimation results upon varying the mean prior on the Calvo price stickiness parameter. For instance, the average annualized rate of inflation varies from 2.81% with a prior of ξ_p of 0.5 to 2.87% with a prior of 0.75. The average frequency of price adjustment ranges from once every 5.66 months with $\xi_p = 0.5$ to once every 6.4 months with

$\xi_p = 0.75$.

As for the estimated policy rule, we find parameters of the policy response to inflation that vary between 2.2 with $\xi_p = 0.5$ to 2.06 with $\xi_p = 0.75$. Therefore, while our estimates suggest that the Fed has adopted a more aggressive stance against fighting inflation in the second subperiod, it nonetheless indicates that monetary policy was more somewhat more accommodative after 1982 under the growth-rule than either the gap-rule or the mixed-rule.

The estimated policy response to output growth is found to be relatively high, between 0.46 and 0.481 depending on the prior on the price stickiness parameter. Note that this parameter under the growth-rule does not seem sensitive to varying the prior on the price stickiness parameter as we found under the mixed-rule.

Based on the estimation results from the growth-rule, that is a trend inflation below 3%, relatively flexible prices, a policy response to inflation above 2.06 and to output growth above 0.46, we find a probability of determinacy of 1 with certainty, and this for the entire range of priors from 0.5 to 0.75.

The marginal data density $\log P(X^T)$ predicted by alternative models suggest the following ranking of models for priors of $\xi_p = 0.5, 0.55, 0.65$: growth-rule model $(-56.36 \mid \xi_p = 0.5; -57.32 \mid \xi_p = 0.55; -57.63 \mid \xi_p = 0.65) >$ mixed-rule model $(-56.9 \mid \xi_p = 0.5; -57.92 \mid \xi_p = 0.55; -59.46 \mid \xi_p = 0.65) >$ gap-rule model $(-68.93 \mid \xi_p = 0.5; -63.44 \mid \xi_p = 0.55; -62.5 \mid \xi_p = 0.65)$.

These findings tell us that the growth-rule model is preferred for the second subperiod. Furthermore, the fact that the gap-rule and mixed-rule models deliver indeterminacy with certainty or near-certainty for the post-1982 period for plausible priors of the price stickiness parameter casts serious doubts as to whether the Fed followed one of these two policy rules during the Great Moderation. By contrast, the determinacy finding with certainty for the full range of mean priors of ξ_p provides evidence supporting the growth-rule model.

6 Conclusion

With the help of a Bayesian estimation method allowing explicitly for the possibility of an indeterminate outcome, we have shown that a policy rule that targets to a minimum the level of the output gap for measure of economic activity was a key reason combined with a high level of trend inflation for indeterminacy during the pre-Volcker years. At the same time, our evidence suggests that a policy rule targeting output growth has the potential to achieve determinacy in a period of high inflation.

Evidence for the post-1982 period confirmed the stabilizing powers of a policy rule targeting output growth. A more hawkish stance against inflation, combined with a stronger policy response to output growth, helped bringing down trend inflation. Taken together, these factors helped achieving determinacy during the Great Moderation.

Our estimation results seem to confirm there was a shift in the policy rule towards abandoning the output gap as a target during the pre-Volcker years in favor of targeting output growth during the Great Moderation. Therefore, studies on postwar business cycle fluctuations, indeterminacy, and sources of the Great Moderation imposing a single policy rule throughout may be misleading. As we did for indeterminacy, this means that the main factors for the transition from the Great Inflation to the Great Moderation need to be reassessed. We plan to undertake this task in the near future.

References

Ascari, G., Phaneuf, L. and Sims, E. R.: 2018, On the welfare and cyclical implications of moderate trend inflation, *Journal of Monetary Economics* **99**(C), 56–71.

Ascari, G. and Ropele, T.: 2009, Trend inflation, taylor principle, and indeterminacy, *Journal of Money, Credit and Banking* **41**(8), 1557–1584.

Bils, M. and Klenow, P. J.: 2004, Some Evidence on the Importance of Sticky Prices, *Journal of Political Economy* **112**(5), 947–985.

Blanchard, O. and Galí, J.: 2007, Real wage rigidities and the new keynesian model, *Journal of Money, Credit and Banking* **39**(s1), 35–65.

Calvo, G.: 1983, Staggered prices in a utility-maximizing framework, *Journal of Monetary Economics* **12**(3), 383–398.

Chari, V. V., Kehoe, P. J. and McGrattan, E. R.: 2009, New Keynesian Models: Not Yet Useful for Policy Analysis, *American Economic Journal: Macroeconomics* **1**(1), 242–266.

Christiano, L. J.: 2015, Comment on "Networks and the Macroeconomy: An Empirical Exploration", *NBER Macroeconomics Annual 2015, Volume 30*, NBER Chapters, National Bureau of Economic Research, Inc, pp. 346–373.

Christiano, L. J., Eichenbaum, M. and Evans, C. L.: 2005, Nominal rigidities and the dynamic effects of a shock to monetary policy, *Journal of Political Economy* **113**(1), 1–45.

Clarida, R., Galí, J. and Gertler, M.: 1999, The science of monetary policy: A new keynesian perspective, *Journal of Economic Literature* **37**(4), 1661–1707.

Clarida, R., Galí, J. and Gertler, M.: 2000, Monetary policy rules and macroeconomic stability: Evidence and some theory, *Quarterly Journal of Economics* **115**, 147–180.

Cogley, T. and Sbordone, A. M.: 2008, Trend inflation, indexation, and inflation persistence in the new keynesian phillips curve, *American Economic Review* **98**(5), 2101–26.

Coibion, O. and Gorodnichenko, Y.: 2011, Monetary policy, trend inflation and the great moderation: An alternative interpretation, *American Economic Review* **101**(1), 341–370.

Galí, J. and Gertler, M.: 1999, Inflation dynamics: A structural econometric analysis, *Journal of Monetary Economics* **44**(2), 195–222.

Haque, Q., Groshenny, N. and Weder, M.: 2021, Do we really know that u.s. monetary policy was destabilizing in the 1970s?, *European Economic Review* **131**, 103615.

Herbst, E. P. and Schorfheide, F.: 2016, *Bayesian Estimation of DSGE Models*, number 10612, Princeton University Press.

Herbst, E. and Schorfheide, F.: 2014, Sequential Monte Carlo Sampling For Dsge Models, *Journal of Applied Econometrics* **29**(7), 1073–1098.

Hirose, Y., Kurozumi, T. and Van Zandweghe, W.: 2020, Monetary policy and macroeconomic stability revisited, *Review of Economic Dynamics* **37**, 255 – 274.

Justiniano, A. and Primiceri, G.: 2008, The time varying volatility of macroeconomic fluctuations, *American Economic Review* **98**(3), 604–641.

Justiniano, A., Primiceri, G. and Tambalotti, A.: 2010, Investment shocks and business cycles, *Journal of Monetary Economics* **57(2)**, 132–145.

Justiniano, A., Primiceri, G. and Tambalotti, A.: 2011, Investment shocks and the relative price of investment, *Review of Economic Dynamics* **14(1)**, 101–121.

Khan, H., Phaneuf, L. and Victor, J. G.: 2020, Rules-based monetary policy and the threat of indeterminacy when trend inflation is low, *Journal of Monetary Economics (forthcoming)* .

Kiley, M. T.: 2007, Is Moderate-to-High Inflation Inherently Unstable?, *International Journal of Central Banking* **3**(2), 173–201.

Lubik, T. A. and Schorfheide, F.: 2003, Computing sunspot equilibria in linear rational expectations models, *Journal of Economic Dynamics and Control* **28**(2), 273–285.

Lubik, T. A. and Schorfheide, F.: 2004, Testing for indeterminacy: An application to u.s. monetary policy, *American Economic Review* **94**(1), 190–217.

Nakamura, E. and Steinsson, J.: 2008, Five Facts about Prices: A Reevaluation of Menu Cost Models, *The Quarterly Journal of Economics* **123**(4), 1415–1464.

Orphanides, A.: 2002, Monetary-policy rules and the great inflation, *American Economic Review* **92**(2), 115–120.

Phaneuf, L., Sims, E. and Victor, J. G.: 2018, Inflation, output and markup dynamics with purely forward-looking wage and price setters, *European Economic Review* **105**(C), 115–134.

Smets, F. and Wouters, R.: 2007, Shocks and frictions in us business cycles: A bayesian dsge approach, *American Economic Review* **97**(3), 586–606.

Taylor, J. B.: 1993, Discretion versus policy rules in practice, *Carnegie-Rochester Conference Series on Public Policy* **39**(1), 195–214.

Walsh, C.: 2003, Speed limit policies: The output gap and optimal monetary policy, *American Economic Review* **93**(1), 265–278.

Wolman, A. L.: 2011, The Optimal Rate of Inflation with Trending Relative Prices, *Journal of Money, Credit and Banking* **43**, 355–384.

Woodford, M.: 2007, Interpreting Inflation Persistence: Comments on the Conference on "Quantitative Evidence on Price Determination", *Journal of Money, Credit and Banking* **39**(1), 203–210.

Table 1: 1966I:1979II POSTERIOR ESTIMATES GAP RULE

	Prior			Posterior		
	Dist.	Mean	SD	$\xi_p = 0.50$	$\xi_p = 0.55$	$\xi_p = 0.65$
h	Beta	0.7	0.1	0.582	0.520	0.538
				[0.449,0.724]	[0.407,0.631]	[0.410,0.667]
ξ_p	Beta	—	0.1	0.564	0.585	0.644
				[0.471,0.658]	[0.496,0.676]	[0.544,0.734]
α_π	Gamma	1.5	0.3	1.408	1.666	1.652
				[1.001,1.851]	[1.285,2.107]	[1.074,2.184]
α_y	Gamma	0.125	0.1	0.280	0.327	0.260
				[0.121,0.432]	[0.187,0.468]	[0.089,0.410]
α_{dy}	Gamma	0.125	0.1	—	—	—
				[—,—]	[—,—]	[—,—]
ρ_R	Beta	0.6	0.2	0.566	0.500	0.524
				[0.414,0.732]	[0.348,0.646]	[0.334,0.694]
$\bar{A}$	Normal	0.370	0.15	0.386	0.356	0.370
				[0.188,0.584]	[0.174,0.519]	[0.151,0.558]
$\bar{\pi}$	Normal	0.985	0.75	1.406	1.566	1.336
				[1.078,1.721]	[1.266,1.885]	[0.970,1.655]
$\bar{R}$	Gamma	1.597	0.25	1.651	1.734	1.582
				[1.382,1.963]	[1.500,2.005]	[1.286,1.899]
ρ_b	Beta	0.5	0.2	0.472	0.567	0.505
				[0.211,0.734]	[0.323,0.798]	[0.190,0.782]
ρ_A	Beta	0.5	0.2	0.630	0.638	0.655
				[0.346,0.879]	[0.460,0.836]	[0.336,0.906]
ρ_r	Beta	0.5	0.2	0.455	0.471	0.525
				[0.256,0.642]	[0.300,0.642]	[0.329,0.697]
σ_b	Inverse Gamma	0.5	4	2.127	2.172	1.487
				[0.287,3.325]	[0.649,3.193]	[0.287,2.708]
σ_A	Inverse Gamma	0.5	4	0.526	0.460	0.561
				[0.246,0.806]	[0.266,0.674]	[0.256,0.820]
σ_r	Inverse Gamma	0.5	4	0.300	0.332	0.344
				[0.233,0.368]	[0.261,0.407]	[0.250,0.447]
σ_ζ	Inverse Gamma	0.5	4	0.352	0.346	0.365
				[0.257,0.433]	[0.263,0.426]	[0.254,0.466]
M_b	Normal	0	1	-0.012	0.046	-0.017
				[-0.216,0.232]	[-0.096,0.184]	[-0.370,0.336]
M_A	Normal	0	1	-0.095	0.205	0.295
				[-0.970,0.901]	[-0.654,0.980]	[-0.836,1.427]
M_r	Normal	0	1	0.019	0.482	0.359
				[-0.696,0.832]	[-0.241,1.061]	[-0.594,1.114]
log p(X^T)				-120.9342	-119.1691	-121.2839
Prob(det)				0.0038	0.0002	0.0017

Notes: In each case above, the column references the prior mean of the Calvo parameter and for each case the prior standard deviation is set to 0.1. Numbers in square brackets indicate 90% confidence intervals.

Table 2: 1966I:1979II POSTERIOR ESTIMATES MIXED RULE

	Prior			Posterior		
	Dist.	Mean	SD	$\xi_p = 0.50$	$\xi_p = 0.55$	$\xi_p = 0.65$
h	Beta	0.7	0.1	0.551	0.567	0.589
				[0.452,0.667]	[0.457,0.685]	[0.453,0.713]
ξ_p	Beta	—	0.1	0.586	0.614	0.638
				[0.481,0.695]	[0.516,0.726]	[0.541,0.722]
α_π	Gamma	1.5	0.3	1.499	1.613	1.495
				[1.015,1.973]	[1.129,2.161]	[1.044,1.911]
α_y	Gamma	0.125	0.1	0.268	0.237	0.282
				[0.091,0.447]	[0.061,0.392]	[0.041,0.541]
α_{dy}	Gamma	0.125	0.1	0.112	0.121	0.120
				[0.003,0.228]	[0.006,0.248]	[0.004,0.241]
ρ_R	Beta	0.6	0.2	0.550	0.577	0.605
				[0.362,0.712]	[0.400,0.761]	[0.419,0.811]
$\bar{A}$	Normal	0.370	0.15	0.363	0.391	0.352
				[0.186,0.525]	[0.205,0.583]	[0.182, 0.531]
$\bar{\pi}$	Normal	0.985	0.75	1.485	1.405	1.392
				[1.217,1.766]	[1.054,1.729]	[1.083,1.695]
$\bar{R}$	Gamma	1.597	0.25	1.690	1.623	1.627
				[1.438,1.945]	[1.343,1.918]	[1.375,1.910]
ρ_b	Beta	0.5	0.2	0.565	0.515	0.506
				[0.323,0.831]	[0.214,0.775]	[0.255,0.764]
ρ_A	Beta	0.5	0.2	0.480	0.655	0.539
				[0.210,0.850]	[0.438,0.898]	[0.253,0.865]
ρ_r	Beta	0.5	0.2	0.454	0.530	0.432
				[0.249,0.639]	[0.326,0.734]	[0.202,0.662]
σ_b	Inverse Gamma	0.5	4	1.972	1.648	2.205
				[0.345,3.017]	[0.268,3.046]	[0.309,3.512]
σ_A	Inverse Gamma	0.5	4	0.607	0.616	0.537
				[0.260,0.946]	[0.270,0.924]	[0.260,0.812]
σ_r	Inverse Gamma	0.5	4	0.311	0.339	0.310
				[0.235,0.382]	[0.245,0.432]	[0.225,0.385]
σ_ζ	Inverse Gamma	0.5	4	0.375	0.379	0.372
				[0.264,0.474]	[0.262,0.476]	[0.274,0.472]
M_b	Normal	0	1	-0.006	-0.004	-0.007
				[-0.311,0.311]	[-0.436,0.326]	[-0.276,0.260]
M_A	Normal	0	1	0.024	0.143	0.065
				[-0.753,0.839]	[-0.849,1.013]	[-0.707,0.901]
M_r	Normal	0	1	-0.007	0.173	0.223
				[-0.853,0.765]	[-0.506,0.839]	[-0.459,0.877]
log p(X^T)				-120.1704	-120.3639	-119.6619
Prob(det)				0.0472	0.0184	0.0000

Notes: In each case above, the column references the prior mean of the Calvo parameter and for each case the prior standard deviation is set to 0.1. Numbers in square brackets indicate 90% confidence intervals.

Table 3: 1966I:1979II POSTERIOR ESTIMATES GROWTH RULE

	Prior			Posterior		
	Dist.	Mean	SD	$\xi_p = 0.50$	$\xi_p = 0.55$	$\xi_p = 0.65$
h	Beta	0.7	0.1	0.586	0.603	0.611
				[0.480,0.693]	[0.496,0.709]	[0.484,0.743]
ξ_p	Beta	—	0.1	0.403	0.442	0.547
				[0.293,0.498]	[0.305,0.614]	[0.371,0.705]
α_π	Gamma	1.5	0.3	1.522	1.452	1.324
				[1.144,1.956]	[0.870,1.892]	[0.784,1.791]
α_y	Gamma	0.125	0.1	—	—	—
				[—,—]	[—,—]	[—,—]
α_{dy}	Gamma	0.125	0.1	0.181	0.182	0.203
				[0.033,0.329]	[0.025,0.315]	[0.025,0.366]
ρ_R	Beta	0.6	0.2	0.439	0.469	0.546
				[0.254,0.597]	[0.277,0.666]	[0.363,0.755]
$\bar{A}$	Normal	0.370	0.15	0.369	0.371	0.379
				[0.125,0.575]	[0.145,0.599]	[0.149,0.599]
$\bar{\pi}$	Normal	0.985	0.75	1.371	1.369	1.354
				[1.139,1.605]	[1.100,1.604]	[1.029,1.664]
$\bar{R}$	Gamma	1.597	0.25	1.611	1.606	1.614
				[1.367,1.864]	[1.344,1.874]	[1.317,1.928]
ρ_b	Beta	0.5	0.2	0.754	0.704	0.585
				[0.640,0.902]	[0.421,0.906]	[0.231,0.884]
ρ_A	Beta	0.5	0.2	0.266	0.351	0.504
				[0.047,0.426]	[0.061,0.709]	[0.176,0.846]
ρ_r	Beta	0.5	0.2	0.481	0.473	0.472
				[0.364,0.612]	[0.326,0.613]	[0.289,0.646]
σ_b	Inverse Gamma	0.5	4	1.048	1.067	1.143
				[0.564,1.511]	[0.362,1.621]	[0.246,2.155]
σ_A	Inverse Gamma	0.5	4	1.897	1.749	1.393
				[1.429,2.453]	[0.547,2.420]	[0.393,2.282]
σ_r	Inverse Gamma	0.5	4	0.375	0.363	0.333
				[0.281,0.470]	[0.251,0.460]	[0.245,0.427]
σ_ζ	Inverse Gamma	0.5	4	0.583	0.531	0.454
				[0.261,0.877]	[0.268,0.814]	[0.249,0.716]
M_b	Normal	0	1	0.124	-0.075	-0.011
				[-1.560,1.664]	[-1.497,1.381]	[-1.179,1.054]
M_A	Normal	0	1	-0.146	0.009	-0.139
				[-1.647,1.340]	[-1.604,1.752]	[-0.914,0.534]
M_r	Normal	0	1	0.110	-0.106	0.162
				[-1.465,1.717]	[-1.678,1.303]	[-0.914,1.825]
log p(X^T)				-123.56	-124.424	-125.8845
Prob(det)				0.9474	0.7967	0.4390

Notes: In each case above, the column references the prior mean of the Calvo parameter and for each case the prior standard deviation is set to 0.1. Numbers in square brackets indicate 90% confidence intervals.

Table 4: 1982IV:2008IV POSTERIOR ESTIMATES GAP RULE

	Prior			Posterior		
	Dist.	Mean	SD	$\xi_p = 0.50$	$\xi_p = 0.55$	$\xi_p = 0.65$
h	Beta	0.7	0.1	0.584	0.536	0.640
				[0.474,0.727]	[0.425,0.652]	[0.538,0.748]
ξ_p	Beta	—	0.1	0.433	0.749	0.766
				[0.284,0.735]	[0.694,0.804]	[0.707,0.828]
α_π	Gamma	1.5	0.3	2.316	2.510	2.588
				[1.941,2.770]	[2.149,2.882]	[2.156,3.001]
α_y	Gamma	0.125	0.1	0.146	0.064	0.066
				[0.004,0.304]	[0.023,0.107]	[0.019,0.116]
α_{dy}	Gamma	0.125	0.1	—	—	—
				[—,—]	[—,—]	[—,—]
ρ_R	Beta	0.6	0.2	0.614	0.591	0.615
				[0.514,0.708]	[0.504,0.677]	[0.529,0.705]
$\bar{A}$	Normal	0.370	0.15	0.389	0.379	0.365
				[0.226,0.554]	[0.220,0.543]	[0.172,0.550]
$\bar{\pi}$	Normal	0.985	0.75	0.705	0.909	0.840
				[0.528,0.873]	[0.750,1.067]	[0.617,1.059]
$\bar{R}$	Gamma	1.597	0.25	1.407	1.627	1.526
				[1.163,1.640]	[1.435,1.843]	[1.240,1.807]
ρ_b	Beta	0.5	0.2	0.879	0.583	0.591
				[0.841,0.952]	[0.310,0.858]	[0.271,0.909]
ρ_A	Beta	0.5	0.2	0.269	0.735	0.718
				[0.066,0.700]	[0.629,0.838]	[0.427,0.903]
ρ_r	Beta	0.5	0.2	0.639	0.758	0.771
				[0.517,0.742]	[0.705,0.807]	[0.711,0.831]
σ_b	Inverse Gamma	0.5	4	1.635	0.644	0.720
				[1.040,2.553]	[0.320,0.989]	[0.210,1.466]
σ_A	Inverse Gamma	0.5	4	1.107	0.487	0.527
				[0.491,1.571]	[0.298,0.672]	[0.291,0.771]
σ_r	Inverse Gamma	0.5	4	0.243	0.259	0.261
				[0.188,0.296]	[0.214,0.303]	[0.202,0.315]
σ_ζ	Inverse Gamma	0.5	4	0.664	0.380	0.404
				[0.225,1.240]	[0.272,0.474]	[0.313,0.498]
M_b	Normal	0	1	0.118	-0.478	-0.126
				[-1.323,1.607]	[-0.972,-0.040]	[-0.636,0.297]
M_A	Normal	0	1	-0.115	-0.317	-0.184
				[-1.651,1.570]	[-0.698,0.075]	[-0.584,0.297]
M_r	Normal	0	1	0.393	2.146	1.987
				[-1.058,2.256]	[1.499,2.767]	[1.367,2.679]
log p(X^T)				-68.9284	-63.4437	-62.4963
Prob(det)				0.8696	0.0017	0.0001

Notes: In each case above, the column references the prior mean of the Calvo parameter and for each case the prior standard deviation is set to 0.1. Numbers in square brackets indicate 90% confidence intervals.

Table 5: 1982IV:2008IV POSTERIOR ESTIMATES MIXED RULE

	Prior			Posterior		
	Dist.	Mean	SD	$\xi_p = 0.50$	$\xi_p = 0.55$	$\xi_p = 0.65$
h	Beta	0.7	0.1	0.573	0.602	0.643
				[0.480,0.669]	[0.511,0.691]	[0.543,0.743]
ξ_p	Beta	—	0.1	0.446	0.483	0.755
				[0.356,0.548]	[0.375,0.579]	[0.677,0.831]
α_π	Gamma	1.5	0.3	2.231	2.122	2.317
				[1.753,2.663]	[1.695,2.560]	[1.788,2.820]
α_y	Gamma	0.125	0.1	0.093	0.092	0.081
				[0.002,0.185]	[0.002,0.185]	[0.016,0.168]
α_{dy}	Gamma	0.125	0.1	0.448	0.478	0.257
				[0.263,0.613]	[0.280,0.654]	[0.008,0.505]
ρ_R	Beta	0.6	0.2	0.609	0.618	0.627
				[0.508,0.711]	[0.521,0.722]	[0.529,0.733]
$\bar{A}$	Normal	0.370	0.15	0.400	0.395	0.361
				[0.241,0.584]	[0.201,0.568]	[0.152,0.559]
$\bar{\pi}$	Normal	0.985	0.75	0.692	0.697	0.844
				[0.542,0.847]	[0.533,0.858]	[0.605,1.071]
$\bar{R}$	Gamma	1.597	0.25	1.418	1.430	1.517
				[1.169,1.688]	[1.188,1.705]	[1.234,1.772]
ρ_b	Beta	0.5	0.2	0.920	0.914	0.564
				[0.885,0.955]	[0.883,0.951]	[0.237,0.924]
ρ_A	Beta	0.5	0.2	0.158	0.173	0.755
				[0.037,0.275]	[0.025,0.304]	[0.618,0.917]
ρ_r	Beta	0.5	0.2	0.590	0.599	0.743
				[0.494,0.676]	[0.495,0.695]	[0.658,0.832]
σ_b	Inverse Gamma	0.5	4	1.744	1.626	0.689
				[1.123,2.417]	[1.125,2.103]	[0.267,1.385]
σ_A	Inverse Gamma	0.5	4	1.276	1.372	0.513
				[0.984,1.549]	[1.042,1.711]	[0.263,0.784]
σ_r	Inverse Gamma	0.5	4	0.237	0.229	0.236
				[0.186,0.288]	[0.180,0.278]	[0.179,0.297]
σ_ζ	Inverse Gamma	0.5	4	0.608	0.587	0.429
				[0.270,0.971]	[0.272,0.896]	[0.263,0.518]
M_b	Normal	0	1	-0.003	-0.038	-0.125
				[-1.588,1.637]	[-1.635,1.568]	[-0.756,0.447]
M_A	Normal	0	1	0.037	-0.069	-0.045
				[-1.630,1.572]	[-1.538,1.545]	[-0.603,0.501]
M_r	Normal	0	1	-0.063	0.137	1.245
				[-1.741,1.627]	[-1.527,1.723]	[0.010,2.445]
log p(X^T)				-56.8982	-57.9228	-59.4630
Prob(det)				1.0000	0.9984	0.0282

Notes: In each case above, the column references the prior mean of the Calvo parameter and for each case the prior standard deviation is set to 0.1. Numbers in square brackets indicate 90% confidence intervals.

Table 6: 1982IV:2008IV POSTERIOR ESTIMATES GROWTH RULE

	Prior			Posterior		
	Dist.	Mean	SD	$\xi_p = 0.50$	$\xi_p = 0.65$	$\xi_p = 0.75$
h	Beta	0.7	0.1	0.569	0.608	0.606
				[0.474,0.659]	[0.518,0.695]	[0.516,0.701]
ξ_p	Beta	—	0.1	0.470	0.528	0.532
				[0.362,0.576]	[0.415,0.637]	[0.419,0.651]
α_π	Gamma	1.5	0.3	2.198	2.080	2.057
				[1.739,2.670]	[1.598,2.549]	[1.633,2.504]
α_y	Gamma	0.125	0.1	—	—	—
				[—,—]	[—,—]	[—,—]
α_{dy}	Gamma	0.125	0.1	0.460	0.481	0.480
				[0.291,0.649]	[0.299,0.670]	[0.287,0.660]
ρ_R	Beta	0.6	0.2	0.601	0.601	0.599
				[0.489,0.703]	[0.497,0.707]	[0.500,0.720]
$\bar{A}$	Normal	0.370	0.15	0.410	0.391	0.396
				[0.243,0.588]	[0.206,0.572]	[0.206,0.571]
$\bar{\pi}$	Normal	0.985	0.75	0.702	0.715	0.717
				[0.543,0.862]	[0.546,0.884]	[0.537,0.881]
$\bar{R}$	Gamma	1.597	0.25	1.435	1.445	1.439
				[1.171,1.698]	[1.206,1.714]	[1.197,1.687]
ρ_b	Beta	0.5	0.2	0.922	0.919	0.921
				[0.892,0.955]	[0.886,0.950]	[0.889,0.955]
ρ_A	Beta	0.5	0.2	0.163	0.188	0.201
				[0.031,0.289]	[0.031,0.338]	[0.034,0.356]
ρ_r	Beta	0.5	0.2	0.573	0.601	0.599
				[0.480,0.670]	[0.503,0.696]	[0.499,0.698]
σ_b	Inverse Gamma	0.5	4	1.703	1.640	1.677
				[1.111,2.237]	[1.089,2.180]	[1.095,2.290]
σ_A	Inverse Gamma	0.5	4	1.293	1.423	1.393
				[0.983,1.579]	[1.064,1.747]	[1.079,1.731]
σ_r	Inverse Gamma	0.5	4	0.232	0.227	0.229
				[0.182,0.280]	[0.177,0.273]	[0.177,0.279]
σ_ζ	Inverse Gamma	0.5	4	0.602	0.615	0.537
				[0.276,0.951]	[0.263,0.961]	[0.293,0.794]
M_b	Normal	0	1	0.026	0.034	-0.041
				[-1.644,1.659]	[-1.486,1.680]	[-1.680,1.573]
M_A	Normal	0	1	-0.011	0.005	-0.039
				[-1.676,1.551]	[-1.603,1.735]	[-1.595,1.373]
M_r	Normal	0	1	0.004	0.033	-0.025
				[-1.545,1.660]	[-1.606,1.646]	[-1.580,1.484]
log p(X^T)				-56.36	-57.6253	-61.1323
Prob(det)				1.0000	1.0000	0.9982

Notes: In each case above, the column references the prior mean of the Calvo parameter and for each case the prior standard deviation is set to 0.1. Numbers in square brackets indicate 90% confidence intervals.

A Data and Variable Construction

Variable name	Source	Notes
Real Gross Domestic Product	FRED	FRED code (GDPC96)
Gross Domestic Product: Implicit Price Deflator	FRED	FRED code (GDPDEF)
Effective Federal Funds Rate	FRED	FRED code (FEDFUNDS)
Working Age Population	FRED	FRED code (LFWA64TTUSM647S)

We define output growth as,

$$100 \log \Delta Y = 100\Bigg(\log \left(\frac{GDPC96_t}{LFWA64TTUSM647S_t} \right) - \log \left(\frac{GDPC96_{t-1}}{LFWA64TTUSM647S_{t-1}} \right) \Bigg),$$

and inflation as,

$$100 \log \Delta P = 100\Bigg(\log GDPDEF_t - \log GDPDEF_{t-1} \Bigg),$$

and the Federal Funds rate (converted to quarterly units),

$$100 \log R_t = 100 \log \left(1 + \frac{FEDFUNDS}{400} \right).$$