Second Order Interest-Smoothing, Time-Varying Inflation Target and The Prospect of Indeterminacy*

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February 2, 2022

Abstract

Single-equation estimation of monetary policy rules highlights second order interest smoothing. We offer evidence based on Bayesian model consistent estimation showing that second order smoothing prevailed prior to 1980 and after 1982 until the Great Recession. We use recently developed methods for solving linear rational expectations models to jointly accommodate determinacy and indeterminacy. Our estimation strategy employs a sequential Monte Carlo algorithm permitting estimation of both cases simultaneously. We estimate several versions of New Keynesian models with positive trend inflation. We find that models with second order smoothing, a time-varying inflation target, and persistent policy shocks predict determinacy prior to 1980 and after 1982. Key to our findings is the use of an observable for the inflation target that dates back to the 1960s. Models with policy rules targeting output growth are statistically preferred to models with rules aiming at the output gap and output growth. We identify several reasons why the previous literature has often concluded to indeterminacy prior to 1980. They include first order interest smoothing rules, no observable for the inflation target, and imposing a fixed inflation target.

JEL classification: E31, E32, E37.

Keywords: Bayesian estimation; Taylor rules; positive trend inflation; second order interest-smoothing; time-varying target inflation; indeterminacy.

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*Phaneuf acknowledges financial support from the Social Sciences and Humanities Research Council for project 435-2020-1061-Achieving Macroeconomic Stability in Economies with Positive Long Run (Trend) Inflation.

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1 Introduction

Conventional wisdom in monetary economics holds that US monetary policy ought to be described by feedback rules stating that the Federal Reserve adjusts the nominal interest rate in response to deviations of inflation and output from their target levels while smoothing short term movements in interest rates, first order smoothing being the key mechanism driving interest rate inertia.

Rudebusch (2002, 2006), however, challenged this view by questioning the source of nominal interest rate inertia. Would the Fed react to persistent variables which are omitted from the policy rule, then persistent policy shocks could account for interest rate inertia without any smoothing intention from the Fed.

Coibion and Gorodnichenko (2012) (CG) reconsider the sources of interest rate inertia. Using data from 1983:I to 2006:IV and a variety of methods, they present evidence supporting second order interest smoothing as the key factor driving interest rate inertia. Their evidence points to second order smoothing parameters which are statistically significant, and autoregressive parameters in the error terms of the policy rules which are either negative or statistically insignificant.

Coibion and Gorodnichenko (2011) import estimates of policy rules featuring second order smoothing into a NK model with positive trend inflation calibrated to US data. Their estimates suggest the Fed practiced a policy that was too accommodative to inflation from 1969 to 1978, a period when trend inflation was high, which resulted into self-fulfilling inflation expectations and indeterminacy. By contrast, during the Great Moderation the Fed adopted a more aggressive stance against inflation, which helped to curb trend inflation and restored determinacy.

Thus far, the evidence about interest smoothing has been based on single equation estimation. Therefore, monetary policy rules have been estimated outside any particular structural macroeconomic model. By contrast, we adopt a Bayesian model consistent approach to monetary policy rules. Then, we assess the implications of model estimates for the Fed's behavior, indeterminacy, and aggregate fluctuations.

Our main contribution in this paper is to show that while second order interest smoothing is supported by Bayesian estimation, our model estimates essentially overturn CG' main conclusion about the type of policy implemented by the Fed, the rule followed, and the prevalence of indeterminacy during the pre-Volcker period. With the help of many alternative policy rule specifications, we also identify a number of reasons why previous studies on indeterminacy have concluded otherwise.

To make our point as simply as possible and emphasize the role played by alternative monetary policy rule specifications in driving our findings, we focus here on standard New Keynesian (NK) models with sticky prices as did most previous research on indeterminacy. Besides second or first order interest smoothing, the rules included in our estimated models allow policy responses to deviations of short-run inflation from a time-varying inflation target (e.g. see Erceg and Levin (2003); Ireland (2007); Aruoba and Schorfheide (2011); Del Negro and Eusepi (2011); Del Negro et al. (2015)).

A novel aspect of our work, however, is the use of an observable for the inflation target in our estimation that goes back to the 1960s. This target observable, based on Aruoba and Schorfheide (2011), is generated by estimating the common factor from two series capturing inflation expectations and actual inflation.[1] Having an observable for the inflation target helps to identify the policy rule parameters, in particular those relating to interest rate smoothing and policy shocks, during the pre-1980s and the Great Moderation. As we later show, accounting for this observable has important consequences for model estimates and the prospect of determinacy.

In our policy rules, the central bank can in principle target different measures of economic activity. As in Coibion and Gorodnichenko (2011) and most of the previous literature on indeterminacy, we allow policy rules to target both the level of the output gap and output growth, a procedure we call mixed output targeting (MOT). The central bank may instead

[1] The inflation target series is strongly correlated with inflation expectations from the Survey of Professional Forecasters. For periods where the target series and data on inflation expectations overlap, the correlation coefficient with one and ten year ahead inflation expectations are 0.94 and 0.96, respectively.

target output growth only – a targeting procedure we call output growth targeting (OGT). While MOT rules have been mostly employed in estimated NK models, Walsh (2003) and Orphanides and Williams (2006) have called for monetary policymakers to respond to output growth rather than to the output gap. Also, CG (2011) have shown that OGT rules increase the likelihood of determinacy relative to others targeting only the output gap. Khan et al. (2020) have shown, based on CG' (2011) estimates of second-order smoothing policy rules, that MOT rules would require implausibly high policy responses to inflation to be consistent with determinacy during the pre-1980s relative to those asked by OGT rules.[2]

A novel methodological aspect of our work is the use of a procedure for estimating our models that combines the solution method to models of indeterminacy proposed by Bianchi and Nicoló (2020) (BN) with the Sequential Monte Carlo (SMC) algorithm of Herbst and Schorfheide (2014). This allows for the joint possibility of determinacy and indeterminacy regions of a model in a single estimation, even when the region between the two is unknown. The advantage of the BN approach relative to others, like those of Lubik and Schorfheide (2004) and Hirose et al. (2020), is that it does not require finding the boundary region for each likelihood evaluation which is more computationally costly than our procedure.[3]

Our substantive findings can be summarized as follows. First, we present simulation results based on a calibrated NK model that convey insights about the boundaries of indeterminacy regions conditioned on second order smoothing MOT and OGT rules. We show that insofar as monetary policy policy reacts to the output gap, interest smoothing will have a negligible impact on determinacy regions. If instead the policy rule targets output growth only, the degree of smoothing will matter for determinacy regions.

Next, we report simulation results where we generate data from our model when it is characterized by determinacy or indeterminacy, and then estimate our model on this data. We show that with sample sizes close to our empirical ones, our estimation method correctly identifies the data generated from a model with determinacy or indeterminacy, with HPD intervals containing the true parameters.

[2] Sims (2013) has shown that targeting output growth is welfare enhancing relative to targeting output gap.

[3] Estimating the same model with identical SMC algorithm parameters using the approach in Hirose et al. (2020), we find that our approach is nearly four times faster (90 minutes compared to 5.5 hours).

We provide estimation results for two subperiods: 1964:I to 1979:II and 1983:I to 2005:I. We show that the model delivering the largest marginal data densities for the pre-Volcker period is one that includes a policy rule with two lags of smoothing, a time-varying inflation target, OGT, and a persistent policy shock. This model is preferred to a model where the MOT rule replaces the OGT rule, and to various models with one smoothing lag included in the policy rule. The second order smoothing model with the OGT rule is also preferred to others for the period 1983:I to 2005:I, suggesting that the Fed consistently followed the same rule during the two subperiods.

We find that the second order smoothing OGT rule model with a persistent monetary policy shock predicts determinacy with near certainty for the pre-Volcker period, while the second order smoothing model with a MOT rule predicts determinacy with high probability. With the OGT rule, the policy shock persistence does not matter for determinacy. By contrast, with the MOT rule we obtain indeterminacy with high probability with a white-noise policy shock. Based on our estimated models, we find no evidence when determinacy is achieved for the pre-Volcker period of a policy that was "highly accommodative" to inflation (Clarida et al. 2000) or "dovish" (Coibion and Gorodnichenko 2011).

Then, we ask why in the previous literature indeterminacy has been the most preferred outcome for the pre-1980s, and also why MOT rules have generally been preferred to OGT rules in estimated NK models. For this, we first replace second order interest smoothing by first order smoothing. However, unlike previous works with first order smoothing rules, we still use an observable for the inflation target to estimate our models. We find that the OGT rule model with a persistent policy shock still is preferred by the criterion of the log marginal data densities. Accounting for the inflation target observable seems to give the OGT rule an advantage over the MOT rule whether interest smoothing is second or first order.

First order smoothing OGT rule models predict determinacy with probability 1 whether the policy shock is persistent or white-noise. By contrast, with first order smoothing, the MOT rule model implies indeterminacy with near certainty if the policy shock is persistent, and determinacy with high probability if the policy shock is white-noise. Therefore, statistical inference about the indeterminacy outcome is sensitive to assuming a persistent or white-noise policy shock with the MOT rule but not with the OGT rule.

We compare macroeconomic fluctuations implied by two of our estimated models. One is our preferred model with two smoothing lags, a time-varying inflation target, OGT and a persistent policy shock. The other is one close to model versions which have been typically used in the literature on indeterminacy, with the exception of the target observable, which has one smoothing lag, a time-varying inflation target, MOT, and a persistent policy shock. The former delivers determinacy and the latter indeterminacy. Relative to the OGT model, the estimated MOT model grossly overstates the volatility of output growth, inflation, and the policy rate during the pre-Volcker period, while understating the variability of the inflation target.

Accounting in the estimation for an observable for the time-varying inflation target plays a key role for our determinacy results with second order smoothing. For if we omit this observable in estimating our NK models, we find that the probability of determinacy is 0.5 or lower with the MOT and OGT policy rules.[4]

Finally, several standard NK models, like those of Lubik and Schorfheide (2004) and Hirose et al. (2020), have assumed a fixed inflation target coinciding with the steady state rate of inflation. These studies have favoured indeterminacy for the pre-Volcker period. When estimating NK models with first order smoothing and fixed inflation targets, we find that models with MOT rules are generally preferred to models with OGT rules by the log marginal data densities, and that these models favour the indeterminacy outcome.

The rest of the paper is organized as follows. Section 2 describes our economic model. Section 3 describes the BN solution method to models with indeterminacy, the SMC estimation algorithm, and the data and priors used in the estimation. Section 4 discusses simulation evidence from our model about the determinacy regions and suitability of our estimation strategy. Section 5 analyses our results with second order interest smoothing. Section 6 analyses estimation results with rules which have either first order interest smoothing, no observable for the time-varying inflation target, or a fixed inflation target, as in much of the previous literature. Section 7 puts our new findings into perspective in the broader literature. Section

[4] We also find that the same is true for policy rules with one lag of smoothing.

8 contains concluding remarks.

2 The Model

Our framework is a standard NK model augmented with positive trend inflation. It includes sticky prices *à la* Calvo (1983). Aggregate fluctuations are driven by shocks to the discount rate, TFP, the time-varying inflation target, the policy rule, and if in a state of indeterminacy, by sunspot shocks.

2.1 Households

The representative household maximizes expected utility over final consumption goods C and labour supply L

$$\underset{C_t, L_t, B_t}{\text{Max}} \ \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t b_t \left[\log(C_t - h\bar{C}_{t-1}) - v \frac{L_t^{1+\eta}}{1+\eta} \right], \tag{1}$$

where β is the subjective discount factor, h the degree of external habit formation, η is the inverse elasticity of labour supply, and b_t is an intertemporal preference shock which follows an AR(1) process given by

$$\log b_t = (1 - \rho_b)\log b + \rho_b \log b_{t-1} + \epsilon_t^b, \tag{2}$$

where ϵ_t^b is i.i.d. $N(0, \sigma_b^2)$. The representative household is subject to the following budget constraint

$$B_t + P_t C_t = R_{t-1} B_{t-1} + W_t L_t + \Pi_t, \tag{3}$$

where B_{t-1} is the stock of nominal bonds that the household enters period t with, W_t is the nominal wage rate, P_t is the price of the final consumption good, R_t is the gross nominal interest rate, and Π_t is profits from ownership of the firms.

2.2 Final Goods Firms

Final goods firms operate in a perfectly competitive environment and package intermediate goods into a final aggregate good, Y_t, sold at price P_t. Their maximization problem is given by

$$\underset{Y_t(i)}{\text{Max}} \; P_t Y_t - \int_0^1 P_t(i) Y_t(i) di, \tag{4}$$

where

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}. \tag{5}$$

$P_t(i)$ and $Y_t(i)$ are prices and quantities of intermediate goods, and ϵ is the elasticity of substitution between intermediate goods. The maximization problem yields the standard downward sloping demand function for intermediate firm i's input, which is a function of its relative price and the price elasticity of demand

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} Y_t, \tag{6}$$

and the aggregate price index is given by

$$P_t = \left(\int_0^1 P_t(i)^{1-\epsilon} \right)^{\frac{1}{1-\epsilon}}. \tag{7}$$

The market clearing condition is given by

$$Y_t = C_t. \tag{8}$$

2.3 Intermediate Goods Producers

Intermediate goods are produced by a continuum of monopolistically competitive firms with a constant returns to scale production function given by

$$Y_t(i) = A_t L_t(i), \tag{9}$$

where A_t is a neutral technology shock common to all firms. Technology evolves according to

$$\log g_{A,t} = (1-\rho_z)\log g_A + \rho_z \log g_{A,t-1} + \epsilon_t^z, \tag{10}$$

where $g_{A,t} \equiv A_t / A_{t-1}$ and ϵ_t^z is i.i.d. $N(0, \sigma_z^2)$.

Intermediate producers minimize total costs each period subject to meeting demand

$$\underset{L_t(i)}{\text{Min}}\; W_t L_t(i), \tag{11}$$

subject to

$$A_t L_t(i) \geq \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} Y_t. \tag{12}$$

The minimization problem yields the following first-order condition

$$MC_t = \frac{W_t}{A_t}, \tag{13}$$

where MC_t is the nominal marginal cost in period t, and since all firms are subject to the same technology shock and nominal wages, marginal costs are the same across all firms. Firms are subject to Calvo pricing. Each period firms face a probability of reoptimizing their price given by $1 - \xi_p$. A firm setting its price optimally in period t maximizes the following discounted expected flow of profits

$$\underset{P_t(i)}{\text{Max}}\ \mathbb{E}_t \sum_{\tau=0}^{\infty} (\beta\xi_p)^{\tau} \frac{\lambda_{t+\tau}}{\lambda_t} \left(\frac{P_t(i)}{P_{t+\tau}} \left(\frac{P_t(i)}{P_{t+\tau}} \right)^{-\epsilon} Y_{t+\tau} - mc_{t+\tau} \left(\frac{P_t(i)}{P_{t+\tau}} \right)^{-\epsilon} Y_{t+\tau} \right), \tag{14}$$

where mc_t is the real marginal cost in in period t and λ_t is the marginal utility of nominal income to the representative consumer in period t. Lastly we denote price dispersion in period t by

$$v_t^p = \int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} di. \tag{15}$$

2.4 Monetary Policy

Monetary policy is set according to an endogenous feedback rule which takes the form

$$\left(\frac{R_t}{R} \right) = \left(\frac{R_{t-1}}{R} \right)^{\rho_{R,1}} \left(\frac{R_{t-2}}{R} \right)^{\rho_{R,2}} \left[\left(\frac{\pi_t}{\pi_t^\star} \right)^{\alpha_\pi} \left(X_t \right)^{\alpha_x} \left(\frac{Y_t}{Y_{t-1}} \right)^{\alpha_{\Delta y}} \right]^{1-\rho_{R,1}-\rho_{R,2}} v_t^r, \tag{16}$$

where R is the gross nominal interest rate, $\pi_t^\star$ is a time-varying inflation target, X_t is the output gap defined as $\frac{Y_t}{Y_t^n}$, and $\frac{Y_t}{Y_{t-1}}$ is the gross growth rate of output. The natural rate of output, Y_t^n, is given by

$$v\left(\frac{Y_t^n}{A_t} \right)^{1+\eta} = \left(\frac{\epsilon - 1}{\epsilon} \right) + vh\left(\frac{Y_t^n}{A_t} \right)\left(\frac{Y_{t-1}^n}{A_t} \right). \tag{17}$$

v_t^r and $\pi_t^\star$ are exogenous processes given by

$$\log v_t^r = \rho_r \log v_{t-1}^r + \epsilon_t^r, \tag{18}$$

$$\log \pi_t^\star = (1-\rho_\pi)\pi + \rho_\pi \log \pi_{t-1}^\star + \epsilon_t^\pi, \tag{19}$$

where ϵ_t^r is i.i.d. $N(0, \sigma_r^2)$ and ϵ_t^π is i.i.d. $N(0, \sigma_\pi^2)$.

2.5 Log-Linearization

Solving the model requires detrending output, which is done by removing trend growth and taking a log-linear approximation of the stationary model around the non-stochastic steady state. The full set of non-linear equations which characterize the equilibrium of the model and the log-linearized model are reported in the Appendix.

3 Model Solution, Estimation, and Data

3.1 Rational Expectations Solution Under Indeterminacy

To solve the Linear Rational Expectations (LRE) model allowing for the possibility of indeterminacy, we use the solution method proposed by Bianchi and Nicoló (2020) (henceforth BN). A standard LRE system can be cast in its canonical form given by

$$\Gamma_0(\theta)s_t = \Gamma_1(\theta)s_{t-1} + \Psi(\theta)\epsilon_t + \Pi(\theta)\eta_t, \tag{20}$$

where s_t is a vector of endogenous variables, ϵ_t a vector of exogenous disturbances, and η_t a vector of one-step ahead forecast errors for the expectational variables in the model. The solution method proposed by BN is to augment the canonical form with additional autoregressive equations, with the number of additional equations being equal to the degree of indeterminacy. These additional equations can be used to provide the "missing" explosive roots. This requires that when the model is characterized by indeterminacy of order p, then p of the auxiliary equations must be explosive. When the model is determinant, the auxiliary equations are not explosive and do not influence endogenous variables in the model. In our case we permit one degree of indeterminacy and augment the LRE system in (20) with one additional equation which is given by

$$\omega_t = \left(\frac{1}{\alpha_{BN}}\right)\omega_{t-1} - \zeta_t + \eta_{f,t}, \tag{21}$$

where ζ_t is a sunspot shock which follows $\zeta_t \sim$ i.i.d. $N(0, \sigma_\zeta^2)$. $\eta_{f,t}$ is an expectational error. BN show that the choice of expectational error does not affect the solution of the model when the correlations between the sunspot shock and fundamental shocks are left unrestricted. We assume that this expecational error is associated with inflation, that is, $\eta_{f,t} = \eta_{\pi,t} = \pi_t - E_{t-1}\pi_t$. Additionally, since the model contains trend inflation, the exact boundary of indeterminacy is unknown. As such we treat α_{BN} as a parameter to be estimated alongside the other structural parameters of the model.

Expanding the state space to include the additional auxiliary equation, the LRE model takes the form

$$\hat{\Gamma}_0(\theta)\hat{s}_t = \hat{\Gamma}_1(\theta)\hat{s}_{t-1} + \hat{\Psi}(\theta)\hat{\epsilon}_t + \hat{\Pi}(\theta)\eta_t, \tag{22}$$

where $\hat{s}_t \equiv (s_t, \omega_t)'$ and $\hat{\epsilon}_t \equiv (\epsilon_t, \zeta_t)'$. The matrices $\hat{\Gamma}_0$, $\hat{\Gamma}_1$, $\hat{\Psi}$, $\hat{\Pi}$ are redefined to include the auxiliary equation. (22) can now be solved using standard methods and the augmented representation contains solutions for the model in both the determinacy and indeterminacy regions (given the parameter requirements discussed above). The BN characterization of equilibrium indeterminacy is equivalent to the characterizations one would get using the methodology of Lubik and Schorfheide (2003, 2004) or Farmer et al. (2015).

3.2 Econometric Strategy

To estimate the posterior distributions of the structural parameters and shocks we use Bayesian estimation. Because the model features regions of determinacy and indeterminacy, posterior densities are potentially multi-modal and standard posterior approximation methods such as the random walk Metropolis Hastings (RWMH) algorithm can often get stuck at local modes and fail to explore the entire posterior distribution. This is driven by the construction of the RWMH algorithm, which relies on highly correlated draws. Instead we employ the Sequential Monte Carlo (SMC) method proposed in Herbst and Schorfheide (2014) and discussed in Herbst and Schorfheide (2016). SMC is an importance sampling algorithm but overcomes the main challenge associated with importance sampling, which is finding good proposal densities, by recursively constructing a sequence of distributions which begins at some easy-to-sample initial distribution (in our case, the prior distributions) and using these distributions as proposal densities in the subsequent stages.

The sequence of distributions are given by

$$\pi_n(\theta) = \frac{[p(Y|\theta)]^{\phi_n} p(\theta)}{\int [p(Y|\theta)]^{\phi_n} p(\theta) d\theta}, \tag{23}$$

where ϕ_n increases from 0 to 1 for $n = 1, \ldots, N^{\phi}$.[5] The sequence of distributions for $\phi_n \in (0, 1)$ are referred to as *tempered posteriors* or *bridge* distributions, and the distribution associated with $\phi_n = 1$ is the approximated posterior distribution. The parameter ϕ_n is referred to as the *tempering schedule* and is determined by

$$\phi_n = \left(\frac{n-1}{N_\phi - 1} \right)^\lambda, \tag{24}$$

where n is the current stage and N_ϕ is the total number of stages. λ determines the shape of the tempering schedule. A value of $\lambda = 1$ implies a linear tempering schedule. For high values of λ, initial bridge distributions will be quite similar to the prior distributions, and bridge distributions will be quite different in the final stages of the algorithm. We use a value of $\lambda = 2$, which is the suggested value by Herbst and Schorfheide (2016). The remaining choices to be made are the number of stages and the number of particles in each stage. We follow Herbst and Schorfheide (2016) and use 200 stages ($N_\phi = 200$). However we opt for a larger number of particles than typically recommended and use 25,000 ($N = 25,000$). The rationale for this is that with the Bianchi and Nicoló (2020) solution approach, likelihood evaluation requires a solution to exist and be unique (including the appended auxiliary equation). However, many of the draws at each stage may be discarded due to: (1) the model being determinant and the auxiliary equation being explosive; or (2) the model being indeterminant and the auxiliary equation being non-explosive. This potentially leads to a non-negligible

[5] Because the SMC is based on recursively computing the bridge densities, the posterior density typically denoted $p(\theta|Y)$ is written as $\pi_n(\theta)$, where the subscript n refers to the bridge density in iteration n.

decline in the number of particles at each stage, which we compensate for by increasing the total number of particles. After initializing the algorithm (i.e., drawing initial particles from the prior distributions) and equalizing the initial weights, the algorithm proceeds in three steps:

1. **Correction**: The correction step reweights particles from the previous stage to areas of parameter space with higher likelihoods. Reweighting occurs according to incremental and normalized weights given by

$$\tilde{w}_n^i = [p(Y|\theta_{n-1}^i)]^{\phi_n - \phi_{n-1}}, \qquad \tilde{W}_n^i = \frac{\tilde{w}_n^i W_{n-1}^i}{\frac{1}{N}\sum_{i=1}^N \tilde{w}_n^i W_{n-1}^i},$$

   for all N particles.

2. **Selection**: The selection step computes the effective sample size (ESS) which is given by $ESS_n = \frac{N}{\frac{1}{N}\sum_{i=1}^N (\tilde{W}_i^n)^2}$. If ESS falls below a threshold use multinomial resampling to resample the particles from support points and weights $\{\theta_{n-1}^i, \tilde{W}_n^i\}$ and equalize the weights such that $W_n^i = 1$. We use the threshold suggested by Herbst and Schorfheide (2016), which resamples when $ESS_n < N/2$. In effect, this step ensures that particles do not become too concentrated, and if a function of the variance of the weights falls below a threshold, then resample.

3. **Mutation**: The mutation steps propagates particles $\{\hat{\theta}_i, W_n^i\}$ using a single step of the RWMH algorithm with appropriate tuning parameters to ensure a reasonable acceptance rate.

The final sampling approximation, $\pi_{N_\phi}(\theta)$, yields the estimated posterior density for the parameters.

Model fit. As noted by Herbst and Schorfheide (2016), the correction step approximates the marginal data density as a by-product without having to compute any additional likelihood evaluations. We use this approximation to rank our models. The approximation is given by

$$\hat{p}_{SMC}(Y) = \prod_{n=1}^{N_\phi} \left(\frac{1}{N} \sum_{i=1}^{N} \tilde{w}_n^i W_{n-1}^i \right). \tag{25}$$

Probability of determinacy. To assess the probability of determinacy our analysis examines the posterior distribution of the parameter α_{BN}. As discussed in the previous section, when the model is characterized by determinacy, the parameter α_{BN} must be strictly greater than 1, such that the appended equation is not explosive and has no impact on the dynamics of the model. Thus our probability of determinacy is computed as

$$\mathbb{P}(\text{Determinacy}) = \frac{1}{N} \sum_{i=1}^{N} \mathbf{1}\{\alpha_{BN,N_\phi}^i > 1\}, \tag{26}$$

where $\mathbf{1}$ is an indicator function which equals 1 if $\alpha_{BN} > 1$ for particle i in the final stage.

3.3 Data

To estimate the parameters of the model we use four U.S. quarterly time series: per capita real GDP growth, GDP deflator based inflation, the Federal Funds rate, and a measure of target inflation from Aruoba and Schorfheide (2011). Our sample periods are dictated by the availability of data on target inflation. The construction of the estimation data is described in detail in the Appendix.

We estimate the model's parameters over two different samples. The first sample corresponds to the pre-Volcker era and spans the periods from 1964:I to 1979:II. The second sample corresponds to the Great Moderation and spans from 1983:I to 2005:I.

The mapping of observables to model variables is given by

$$\begin{bmatrix} 100\Delta \log Y_t \\ 100\Delta \log P_t \\ 100 \log R_t \\ 100\Delta \log P_t^\star \end{bmatrix} = \begin{bmatrix} \bar{g}_A \\ \bar{\pi} \\ \bar{r} \\ \bar{\pi} \end{bmatrix} + \begin{bmatrix} \tilde{y}_t - \tilde{y}_{t-1} + \epsilon_t^z \\ \tilde{\pi}_t \\ \tilde{r}_t \\ \tilde{\pi}_t^\star \end{bmatrix}, \tag{27}$$

where $\bar{g_A}$, $\bar{\pi}$, and $\bar{r}$ are the steady state values of the growth rate of output, inflation, and the nominal interest rate, respectively. These values are expressed in net terms given by $100(\bar{g_A} - 1)$, $100(\bar{\pi} - 1)$, and $100(\bar{r} - 1)$.

3.4 Calibration and Prior Distributions

All parameters are estimated except two which are calibrated. We set the elasticity of substitution between differentiated goods to 9, implying a 12.5% price markup with zero trend inflation. The Frisch elasticity of labour supply is set to 1. These values are standard in the literature. The remaining structural parameters are estimated. Priors for the remaining structural parameters and shock processes are presented in Table 1.

The parameter governing consumer habit formation has a beta distribution prior with a mean of 0.7 and standard deviation of 0.1. As in Justiniano et al. (2011), the prior for the Calvo price stickiness parameter is given by a beta distribution with a mean of 0.66 and standard deviation of 0.1.

Priors for the average rate of inflation, growth rate of output, and nominal interest rate are set at roughly their average sample values for the post-WWII period. We use relatively diffuse priors around these means. The average rate of inflation, output growth, and nominal interest rate have normal distribution priors with means of 0.8, 0.4, and 1.2, with standard deviations of 0.75, 0.2, and 0.4, respectively.

For the response parameters governing monetary policy, we generally follow those specified in Smets and Wouters (2007). The response parameters of inflation, the output gap, and output growth have normal distribution priors with means of 1.5, 0.125, and 0.125 with prior standard deviations of 0.3, 0.05, and 0.05. Our monetary policy rule features interest rate smoothing of order two. For these two parameters we rely closely on the empirical estimates in Coibion and Gorodnichenko (2011). For the order one interest rate smoothing parameter, we specify a normal distribution prior with a mean of 1.1 and standard deviation of 0.25. For interest rate smoothing of order two, we specify a normal prior with mean of 0 and standard deviation of 0.25.[6] When monetary policy is characterized by smoothing of order one, we

[6] It is worth noting that with these priors the sum of autoregressive coefficients could yield a rule which is

specify a beta prior with a mean of 0.6 and standard deviation of 0.2.

The exogenous disturbances have standard priors. Shocks have Inverse Gamma priors with means of about 0.6 and standard deviations of 0.3. Persistence parameters of shocks, including persistence of the time-varying inflation target, have beta distributions with means of 0.5 and standard deviations of 0.2. These priors are identical to those chosen by Del Negro et al. (2015).

For the parameter α_{BN}, which determines whether the model is characterized by indeterminacy, we use a Uniform prior over the range $[0.5, 1.5]$ as suggested by Bianchi and Nicoló (2020). Lastly, for the correlation between sunspot shocks and fundamental shocks, we use Uniform priors over the range $[-1, 1]$.

4 Determinacy Regions and Simulated Data

This Section first identifies the determinacy regions with second order interest-smoothing for levels of annualized trend inflation ranging from 0% to 8%. Coibion and Gorodnichenko (2011) show that targeting the output gap increases the prospect of indeterminacy as trend inflation rises relative to targeting output growth. However, when examining these cases, they do not combine output targeting with interest-smoothing. Also, when looking at the impact of interest-smoothing on the determinacy outcome, they limit their analysis to first order smoothing.

4.1 Determinacy Regions

Using numerical simulations we identify the minimum policy responses to inflation consistent with determinacy. We compare "high" interest smoothing which is broadly consistent with CG' (2011) estimates (first order smoothing degree of 1.35 and a second order degree of -0.4), with "mild" smoothing (first order smoothing degree of 0.75 and a second order degree of -0.4).

Figure 1 identifies the policy responses to inflation consistent with determinacy for a rule

super-inertial. However, we discard draws where the sum of coefficients is greater than one.

with second order smoothing and MOT (i.e. a mixed rule targeting both the level of the output gap and output growth). The parameter response to the output gap is set at 0.2. The determinacy regions are traced for high and mild smoothing parameters. The Calvo price stickiness parameter, ξ_p, takes values of 0.55, 0.66 and 0.75. Other parameters are kept at their mean prior values. Figure 2 displays the determinacy regions with a second order interest smoothing policy rule and OGT (i.e. a rule targeting output growth) with a response coefficient on output growth of 0.2.

Under the policy rule with MOT, we find that interest smoothing, whether it is high or mild, is irrelevant for the determinacy outcome. That is, the prospect of determinacy is in this case primarily driven by the policy response to the output gap which has a disproportionate influence over output growth. With relatively flexible prices, or $\xi_p = 0.55$, determinacy can be achieved with a policy reaction to inflation ranging from 1 to 1.22 for an annualized inflation trend going from 0% to 8%. With $\xi_p = 0.66$, the policy response to inflation consistent with determinacy goes from 1 to 1.65 for a similar range of trend inflation rates, while with $\xi_p = 0.75$, the response to inflation must reach 2 to be consistent with determinacy for an inflation trend slightly below 6%.

Under a policy rule with OGT, interest smoothing matters for determinacy. With a high degree of smoothing, determinacy is achieved with a policy reaction to inflation slightly above 1 for annualized rates of trend inflation between 0% and 8%, and this for the three values of ξ_p. With mild smoothing and $\xi_p = 0.55$, determinacy is still achieved with a response to inflation slightly above 1 for trend inflation rates between 0% and 8%. With $\xi_p = 0.66$, the policy response to inflation departs from 1 as trend inflation exceeds 5%, while with $\xi_p = 0.75$, it does so as trend inflation reaches 2.8%.

Therefore, with a policy rule including second order interest smoothing which targets output growth in an economy with positive trend inflation, determinacy will generally be achieved with significantly weaker response to inflation compared to a rule that aims at the output gap. This will be important later when we analyze our estimation results and how MOT and OGT affect the determinacy outcome.

4.2 Simulated Data

To ensure that our estimation strategy is capable of delivering accurate structural parameter estimates and posterior probabilities of determinacy, we conduct the following simulation exercise. First, we calibrate the structural parameters of the model such that in one case the model is characterized by determinacy, while in the other it is characterized by indeterminacy. We generate a sequence of 5,000 normally distributed shocks, with standard deviations given by the calibrated values in Table 2. We then iterate the shocks through the model, keeping the final 75 observations for output growth, inflation, the nominal interest rate, and the inflation target.[7] In both cases we iterate the sunspot shocks through the model, but in the case of determinacy these shocks have no impact on other endogenous variables.

Using the simulated data for the 4 observables, we then estimate the model parameters and probability of determinacy using this data. The estimation results along with the calibrated parameter values used in generating the data are reported in Table 2. The prior distributions for structural parameters are the same as those described in Section 3.4.

Two points are worth emphasizing. First, our simulation evidence suggests that even with only 75 observations, the structural parameters are estimated quite well. In almost all cases, the 90% HPDI interval includes the true parameter value. Second, the simulation reveals that our estimation accurately identifies data generated from models with indeterminacy or determinacy. That is, α_{BN} converges to the correct region. While the simulation exercise benefits from no model uncertainty (the data is drawn from the correct model), it does provide some validation to our results in the rest of the paper.

5 Estimation Results

This Section presents our main estimation results. We first present estimates for the period 1964:I-1979:II from a model with two interest smoothing lags and a time-varying inflation target, allowing a comparison between MOT and OGT policy rules, with and without persistent

[7]Our choice of 75 observations is motivated by our empirical sample sizes which are 62 for the pre-1979 period and 89 for the post-1983 period.

policy shocks. We follow with estimates for the period 1983:I-2005:I. We provide estimates of the structural parameters and shocks with their 90% HPD intervals. The $\log p(X^T)$ represents the marginal data density of a model, while *Prob(det)* is the posterior probability of equilibrium determinacy implied by the model estimates.

5.1 1964:I to 1979:II

Table 3 presents four sets of results for the period 1964:I to 1979:II. The first column reports estimates of a model with a policy rule including second order smoothing, a time-varying inflation target, MOT, and a persistent policy shock. The second column presents estimates of a model where MOT is replaced by OGT. Columns 3 and 4 provide estimates of the MOT and OGT rule models with white-noise policy shocks. All models are estimated with four observables including that for the inflation target.

The second to last row of the table reveals the following ranking of models based on the log marginal data densities $\log p(X^T)$: 1) OGT with AR(1) policy shock $(-91.7) > 2)$ MOT with AR(1) policy shock $(-93.1) > 3)$ OGT with white-noise policy shock $(-95.568) > 4)$ MOT with white-noise policy shock (-95.569). Therefore the OGT rule model is preferred to the MOT rule model by the criterion of data fitting, whether policy shocks are persistent or white-noise.

When looking at the implications of alternative estimated models for indeterminacy, we find that the OGT rule model with a persistent policy shock predicts determinacy with probability .993, while the MOT rule model generates determinacy with probability .849. Interest smoothing is mild in both the MOT and OGT rule models, at .83 and .81 respectively for the first smoothing lag, and $-.32$ and $-.34$ for the second smoothing lag. The first order smoothing parameters are smaller than those reported by Coibion and Gorodnichenko (2011, 2012), while the second order smoothing parameters are more in-line with their estimates.

For MOT and OGT models with persistent policy shocks, the Calvo price stickiness parameter is .62, meaning that the estimated average waiting time between price adjustment is 7.9 months. The estimated average annualized rate of inflation is 4.27 percent in the MOT rule model and 4.17 percent in the OGT rule model.

Given our analysis of determinacy regions with second order interest smoothing based on Figures 1 and 2, with mild interest smoothing, mildly rigid prices and a trend inflation rate of 4% to 5%, determinacy is likely achieved with a policy response to inflation of about 1.22 under the MOT rule and 1 under the OGT rule. Given that the mean policy responses to inflation is 2.0 in the MOT rule model and 2.13 in the OGT rule model, our estimates indicate that the data should normally favour the determinacy outcome, which happens to be the case.

Our results with second order smoothing, a time varying inflation target, and persistent policy shocks run counter the mainstream view that the pre-Volcker period was characterized by indeterminacy, a highly accommodative Fed's policy to inflation, and self-fulfilling inflation expectations. They also run counter to the fact the Fed followed a policy rule targeting a mix of the output gap and output growth.

For persistent policy shock models, the estimated parameter of the AR(1) policy shock process is .38 for the MOT rule model and .37 for the OGT rule model. While moderate, the estimated HPD interval for AR(1) parameters of the policy shock do not contain zero. Therefore, unlike the evidence reported by CG, our estimation results confirm both the significance of second order interest smoothing and mildly persistent monetary policy shocks in the MOT and OGT rule models.

How important is it to account for persistent policy shocks? The third and fourth columns of Table 3 assess the consequences for determinacy of relaxing the assumption of persistence in the policy shock. Assuming white-noise policy shocks significantly affects the determinacy outcome in the MOT rule model, as the probability of determinacy then drops to .33. In this case, the estimated Calvo price stickiness parameter increases to .7 while the mean policy response to inflation drops to 1.4. Furthermore, the estimated average annualized rate of inflation is somewhat higher at 4.6 percent.

By contrast, the estimated probability of determinacy in the OGT rule model with a white-noise policy shock is 1. The response to inflation in the OGT rule departs quite significantly from the response in the MOT rule model at 2.09. The estimated Calvo parameter is .577, meaning more flexible prices on average than with a persistent policy shock. Finally, with white-noise policy shocks, the estimated first order interest smoothing parameters are significantly higher in both the OGT and MOT rule models than those obtained with persistent policy shocks. Therefore, there seems to be a tradeoff between the estimated degree of interest smoothing and degree of persistence in the policy shock.

5.2 1983:I to 2005:I

Table 4 presents the estimation results for the period 1983:I to 2005:I. The second to last row of the table reveal the following ranking of models based on the log marginal data densities $\log p(X^T)$: 1) OGT with AR(1) policy shock $(66.3) > 2)$ MOT with AR(1) policy shock $(61.5) > 3)$ OGT with white-noise policy shock $(38.5) > 4)$ MOT with white-noise policy shock (36.2). Therefore the OGT rule model with a persistent policy shock is still preferred by the criterion of model fit.

When looking at the determinacy outcome, we find that both the OGT and MOT rule models with persistent policy shocks predict determinacy with probability 1. In the case of the MOT rule model, if instead the policy shock is white-noise, the estimated probability of determinacy drops to .06. Meanwhile, the OGT rule model still delivers determinacy with probability 1.

Therefore, our evidence confirms the empirical relevance of models with two smoothing lags, a time-varying inflation target and a persistent policy shock prior to 1980 and after 1982.

6 First Order Interest Smoothing and the Inflation Target: The Pre-Volcker Period

The previous literature on indeterminacy has generally focused on NK models with first order interest smoothing policy rules. With a few exceptions discussed below, it has come to the conclusion that the U.S. economy experienced indeterminacy during the pre-Volcker period. We now ask why indeterminacy has generally been preferred to determinacy, and MOT rules to OGT rules. We look at the importance of using a time-varying inflation target and an observable for the target in the estimation. Furthermore, we assess the implications of estimated first and second order interest smoothing models for macroeconomic fluctuations.

6.1 First Order Smoothing with Time-Varying Inflation Target

We estimate NK models with first order interest smoothing rules and a time-varying inflation target, with and without policy shock persistence. Unlike the previous literature on indeterminacy, we estimate first order smoothing rule models with the help of an observable for the inflation target. For the sake of brevity, Panel A of Table 5 reports only the log marginal data densities and probabilities of determinacy implied by the estimated first order smoothing rule models for the pre-1979 period.

The ranking of first order smoothing rule models based on the log marginal data densities $\log p(X^T)$ is: 1) OGT with AR(1) policy shock $(-93.6) > 2)$ MOT with AR(1) policy shock $(-94.1) > 3)$ OGT with white-noise policy shock $(-98.17) > 4)$ MOT with white-noise policy shock (-99.0). The OGT rule model still is preferred to the MOT rule model. However, the second order smoothing model with the OGT rule and a persistent policy shock is found to be more suitable than first order smoothing models based on the log marginal data densities.

OGT rule models with first order smoothing predict determinacy with probability 1, whether the policy shock is white-noise or persistent. By contrast, with the MOT rule determinacy is very sensitive to policy shock persistence. When assuming a persistent policy shock, we find that the probability of determinacy is .01, while with a white-noise policy shocks this probability increases to .775.

6.2 Time-Varying Inflation Target Without an Observable for the Target

We assess the importance of combining the time-varying inflation target with an observable for the target for our main findings. We reestimate the NK models with second order interest smoothing, a time-varying inflation target, MOT and OGT, and persistent policy shocks, but without the observable for the target.

Panel B of Table 5 presents the estimated probability of determinacy and AR(1) parameters of the target inflation processes. We keep the priors that were previously assigned to the structural parameters and shock processes. Hence, the AR(1) parameter of the inflation target process is assigned a prior mean value of 0.5 and standard deviation of 0.2 (e.g. see Del Negro et al. (2015)).

With standard priors, but without the target observable, the estimated probability of determinacy drops to .51 in the MOT rule model and .42 in the OGT rule model. With the target observable, the estimated AR(1) parameters of the target processes were .92 and .93 in the MOT and OGT rule models. Without the observable they are .49 and .5. Therefore, without the target observable, the estimated AR(1) parameters of target processes tend to be very close to the assigned mean prior likely indicating they are not identified, which is not the case if the target observable is used in the estimation.

Haque (forthcoming) arrives at a different result when estimating a small scale NK model with one smoothing lag in the policy rule, a time-varying inflation target and no target observable. That is, he obtains determinacy with probability 1. However, for this, he assigns a tight prior on the AR(1) parameter of the inflation target process with a mean of 0.95 and standard deviation of 0.025. His estimate for this parameter is .96.

When we assign a persistent prior with mean .95 and standard deviation .025 to the AR(1) parameter in the inflation target process, we obtain probabilities of determinacy equal to 1 for both the MOT and OGT rule models with AR(2) smoothing and persistent policy shocks, and no inflation target observable.[8] Again, without the target observable the estimated AR(1) parameters of the target processes in both models are about equal to the assigned mean prior at about .95.

6.3 First Order Smoothing with Fixed Inflation Target

Lubik and Schorfheide (2004) and Hirose et al. (2020) estimate small scale NK models with a fixed inflation target coinciding with the steady state rate of inflation. These studies favour indeterminacy for the pre-Volcker period. We estimate first order interest smoothing MOT and OGT rule models with fixed inflation targets and policy shocks which are either persis-

[8] In addition Haque (forthcoming) only permits values of σ_π in the range of [0,0.15] with a uniform prior. When we use the persistent prior we also use the uniform prior for the volatility of inflation target shocks.

tent or white-noise. We are able to replicate previous results reported in the literature on indeterminacy.

Panel C of Table 5 reports the estimated log marginal data densities $\log p(X^T)$ and probabilities of determinacy implied by these different models. The ranking of first order smoothing rule models with fixed inflation targets based on the log marginal data densities $\log p(X^T)$ is: 1) MOT with AR(1) policy shock $(-128.0) > 2)$ OGT with AR(1) policy shock $(-130.26) >$ 3) MOT with white-noise policy shock $(-132.79) > 4)$ OGT with white-noise policy shock (-134.67). The determinacy probabilities for these models are .01, .48, .0004 and .0004, respectively.

These findings are consistent with the mainstream view that the US economy experienced indeterminacy during the pre-Volcker period and that a rule targeting both the level of the output gap and output growth seems preferable to one targeting only output growth. However, as we have shown, this is obtained only at the cost of estimating first order interest smoothing models without time-varying inflation target, and without using the inflation target observable in the estimation.

6.4 Macroeconomic Fluctuations

While appropriate estimation of the Taylor rule is important, one could argue that what really matters for policy makers is the implications of estimated models for business cycle analysis. Fo the sake of brevity, we compare the volatility statistics for our four observables implied by two models with persistent policy shocks.

The first model is our preferred one with second order smoothing, a time-varying inflation target, OGT, and a persistent policy shock which is estimated with an observable for the inflation target. This model implies determinacy with a near 1 probability. The second and alternative model is one with first order smoothing, a time-varying inflation target, MOT, and a persistent policy shock also estimated with a target observable. This model predicts indeterminacy with near 1 probability. The later is representative of models used in the previous literature on indeterminacy, except for the use of the target observable.

Table 6 reports the volatility statistics of our four observables predicted by both estimated models for the two subsamples. The first two columns report the standard deviations in the data of the four observables for the periods 1964:I-1979:II and 1983:I-2005:I. The next two coulums report those predicted by our preferred model with OGT. The last two columns present those from the model with first order smoothing and MOT.

The most significant differences between the two models are for the pre-Volcker period. Relative to our preferred model, the alternative model grossly overestimates the volatility of output growth, inflation and of the policy rate, while it underestimates the volatility of the inflation target. This can be explained by the fact that our preferred model is consistent with determinacy during the pre-Volcker period, while the alternative model implies indeterminacy. Note that both models imply relatively similar volatilities of the observables for the second subsample and predict a drop in the volatility of output growth and inflation.

7 Related Literature

This Section discusses how our main findings compare with those reported in the broader literature on monetary policy rules and indeterminacy. The previous literature has often concluded that the US economy experienced indeterminacy during the 1960 & 1970s and determinacy after 1982. These studies have typically assumed policy rules with first order interest smoothing and responses to the output gap and output growth.[9] Policy rules have also included policy responses to deviations of inflation from a fixed target or from a time-varying target with no target observable used in the estimation. Policy shocks have been either white-noise or persistent.

Clarida et al. (2000) (CGG) explain periods of indeterminacy and determinacy using estimates of policy rules with first order smoothing where the response parameters to inflation are lower than 1 during the pre-Volcker period and hence leading to indeterminacy, and higher than 2 after 1982 and resulting into determinacy.

Using a NK price setting model with zero trend inflation estimated with a Bayesian

[9] An exception is Arias et al. (2020), who assume second order interest smoothing, a fixed inflation target, policy responses to output gap and output growth, and a white-noise policy shock in a NK model. They do not address indeterminacy but rather look at whether a trend inflation rate of 4% would represent a threat to determinacy using data from 1984:I to 2008:II.

method that permits the possibility of determinacy or indeterminacy, Lubik and Schorfheide (2004) corroborate CGG's findings that monetary policy was highly accommodative during the pre-Volcker period and conclude that the US economy was in a state of indeterminacy. Compared to our preferred policy rule, theirs includes first order interest smoothing, responses to deviations of inflation and output from target levels, and a white-noise policy shock. The model is estimated using three observables that are HP detrended real GDP, CPI-U inflation, and the average federal funds rate.

Hirose et al. (2020) extend the work of Lubik and Schorfheide by estimating different versions of a NK price setting model with positive trend inflation. They use a full-information Bayesian method and a SMC algorithm which assess regions of determinacy and indeterminacy in a single estimation. Their model includes a policy rule with first order smoothing, responses to deviations of inflation from steady state, to the output gap and output growth. The policy shock obeys an AR(1) process. Three observables are used in the estimation: real GDP growth, inflation, and the federal funds rate. They conclude that the pre-Volcker years were characterized by indeterminacy, while the economy experienced determinacy after 1982. They argue, in line with Coibion and Gorodnichenko (2011), that a more active response to inflation was not sufficient to explain U.S. macroeconomic stability after 1982, and that a lower level of trend inflation, a weaker response to the output gap and a stronger response to output growth also contributed to achieve determinacy.

Unlike Lubik and Schorfheide (2004) and Hirose et al. (2020), our estimations reveal that the model preferred by the criterion of marginal data densities is one with a policy rule including second order interest smoothing, a time-varying inflation target and a reaction to output growth only, and this when a target observable is used in the estimation. This model predicts determinacy rather than indeterminacy. To confirm that our rule specification plays an important role in driving our results, we have also shown consistent with the evidence presented in Lubik and Schorfheide and Hirose et al., that indeterminacy obtains if the policy rule assumes first order smoothing, a time-varying inflation target or a fixed target, and responses to the output gap and output growth.

Bilbiie and Straub (2013) augment the standard NK price setting model with limited asset market participation. They argue that the pre-1980s were characterized by low asset market participation implying that lower interest rates had contractionary rather than expansionary effects on the economy, which led to determinacy during the pre-Volcker years. Compared to our policy rules, theirs features one lag of interest-smoothing, responses to one-period ahead expected inflation and actual output, and a white-noise policy shock. Trend inflation is zero.[10]

Coibion and Gorodnichenko (2011) import single-equation estimates of policy rules featuring two interest-smoothing lags in a calibrated NK model with sticky prices. They conclude that high positive trend inflation and a "dovish" monetary policy led to indeterminacy prior to 1980, while determinacy was achieved during the Great Moderation by lowering the level of trend inflation and adopting a "hawkish" policy stance against inflation.[11] While our evidence is consistent with second order interest-smoothing, it also confirms the significance of mildly persistent policy shocks. Furthermore, our evidence points to determinacy during the pre-Volcker period.

Arias et al. (2020) assume second order interest smoothing as we do. They assume a fixed inflation target, policy responses to output gap and output growth, and a white-noise policy shock. While they confirm the statistical significance of second order smoothing using a Bayesian estimation approach, they do not question indeterminacy prior to 1980, but rather look at whether an inflation trend of 4% would threaten determinacy assuming post-1983 estimates.

Nicoló (2020) estimate the Smets and Wouters (2007) model with zero trend inflation using a Bayesian estimation method allowing him to test whether determinacy or indeterminacy is statistically preferred by the data (Bianchi and Nicoló 2020). He uses a policy rule with first order interest smoothing, a response to deviations of inflation from steady state, responses

[10] Ascari et al. (2017) show that a small amount of nominal wage stickiness will normally prevent inversion of the slope of the IS curve when accounting for limited asset market participation, and hence will invalidate the Inverted Taylor Principle.

[11] Ascari and Ropele (2009) show that with positive trend inflation, determinacy will require policy responses to inflation stronger than dictated by the original Taylor Principle. Khan et al. (2020) show that a policy rule targeting the output gap makes determinacy very unlikely if average (trend) annualized inflation reaches 4 percent.

to the output gap and output growth, and a persistent policy shock. His results favour indeterminacy for samples of data prior to 1980. We also obtain an indeterminacy result for a similar rule used in the estimation.

Haque et al. (2021) estimate a NK price setting model with positive trend inflation, commodity price shocks and a real wage rigidity.[12] Their policy rule includes first order smoothing, responses to deviations of current inflation rates from a fixed inflation target, MOT, and a white-noise policy shock. When combining all these ingredients, they find that determinacy can be achieved with a low response to the output gap. We also report evidence of high probability of determinacy (close to 0.8) based on an estimated model with first order smoothing, MOT, and a white-noise policy shock. But relative to Haque et al. (2021), our determinacy result is due to the observable for the inflation target used in the estimation. Furthermore, our determinacy result does not require the policy response to the output gap to be zero.

Haque (forthcoming) argues that adding a time-varying inflation target to an otherwise standard NK sticky price model with a first order smoothing rule, MOT, and a white-noise policy shock is sufficient to restore determinacy prior to 1980. He does not use an observable for target inflation in the estimation. When assuming standard priors for the persistent inflation target process, and omitting the inflation target as an observable, we find that the probabilities of determinacy for the MOT and OGT models with second order interest rate smoothing drop to .51 and .42, respectively. But when assuming the same tight priors on persistence and standard deviations of the inflation target process as Haque does, we obtain determinacy with probability 1.

8 Conclusion

Previous work based on single-equation estimation has emphasized the role of second order interest smoothing in explaining interest rate inertia and indeterminacy prior to 1980 (Coibion and Gorodnichenko 2011, 2012). With the help of a Bayesian model consistent esti-

[12] They use a form of real rigidity suggested by Blanchard and Galí (2007), who mention that this form real wage rigidity is “an admittedly ad hoc but parsimonious way of modeling the slow adjustment of wages to labor market conditions”.

mation approach to Taylor rules permitting the joint possibility of determinacy and indeterminacy, we have offered new evidence supporting second order interest rate smoothing and the prevalence of determinacy prior to 1980 and after 1982.

We have explored the effects of different Taylor rule specifications in estimated NK models and assessed their implications for the prospect of determinacy. We have compared estimation results and determinacy outcomes for estimated models with policy rules embedding first vs second order interest smoothing, time-varying vs fixed inflation target, MOT vs OGT, and persistent vs white-noise monetary policy shocks.

We have shown that the model preferred by the criterion of marginal data densities is one including second order interest smoothing, a time-varying inflation target, OGT and a persistent policy shock. An observable for the inflation target was used to estimate the model. This model delivered determinacy with near one probability during the 1960 & 1970s. It was followed by a similar model, but with MOT replacing OGT, that implied determinacy with a very high probability for the pre-Volcker period. Models with second order smoothing were also preferred by the same criterion to models with one smoothing lag.

We also compared macroeconomic fluctuations generated by our preferred model with those of a first order interest smoothing model with time-varying inflation, MOT, and a persistent monetary policy shock. We made this comparison because the latter model is representative of models previously used in the literature on indeterminacy. We have found that relative to our preferred model, the alternative model overestimated the volatility of output growth, inflation and the policy rate, while it underestimated the volatility of the inflation target for the pre-Volcker period.

We believe our empirical findings make a strong case for using higher order smoothing rules with a time-varying inflation target, OGT and persistent policy shocks with an observable for the inflation target when undertaking business cycle and policy analyses.

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A Data construction

To estimate the structural parameters of our model, we use four observable. These observable correspond to per capita output growth, inflation, the federal funds rate, and a measure of target inflation. Our first three observable were downloaded from the Federal Reserve Bank of St. Louis on September 28th, 2021. The exact variables and corresponding FRED codes are

- Gross Domestic Product (`GDP`)
- Gross Domestic Product Implicit Price Deflator (`GDPDEF`)
- Effective Federal Funds rate (`FEDFUNDS`)
- Population Level (`CNP16OV`)

Prior to defining per capita GDP, we fit the population level with a Hodrick Prescott filter with a smoothing parameter of 10,000 and use the trend from this series as our measure of population. The rationale for this, as noted by Pfeifer (2020), is that population levels are periodically updated due to censuses or benchmarking in the Current Population Survey. These updates cause spikes in population growth rates not related to changes in the actual population.

Our measure of target inflation is from Aruoba and Schorfheide (2011). The authors estimate this measure by combining three inflation expectation measures in a small state space model and extracting the common factor using the Kalman filter. The series is available for download at Frank Schorfheide's website `https://web.sas.upenn.edu/schorf/publications/` from the paper *Sticky Prices versus Monetary Frictions: An Estimation of Policy Trade-offs*. The data file is titled "inflation-target" and we use the data under the column heading *filtered f0*.

Our observables are then defined as

$$100 \times \Delta \log Y_t = 100 \times \Delta \log \left(\frac{GDP}{GDPDEF \times \hat{POP}} \right), \tag{28}$$

$$100 \times \Delta \log P_t = 100 \times \Delta \log \left(\frac{GDPDEF}{GDPDEF_{-1}} \right), \tag{29}$$

$$100 \times \log R_t = 100 \times \log \left(1 + \frac{FEDFUNDS}{400} \right), \tag{30}$$

$$100 \times \Delta \log P_t^{\star} = \left(\frac{\text{filtered f0}}{4} \right), \tag{31}$$

where $\hat{POP}$ is the filtered population level.

B Full Set of Non-linear Equilibrium Conditions

Below we describe the full set of equations which characterize the equilibrium of the model. There are 17 equations and 17 endogenous variables.

$$\lambda_t P_t = \frac{b_t}{C_t - hC_{t-1}} \tag{32}$$

$$1 = \beta \mathbb{E}_t \left[\left(\frac{\lambda_{t+1}}{\lambda_t} \right) R_t \right] \tag{33}$$

$$b_t v L_t^{\eta} = \lambda_t W_t \tag{34}$$

$$mc_t = \frac{w_t}{A_t} \tag{35}$$

$$C_t = Y_t \tag{36}$$

$$Y_t = \frac{A_t L_t}{v_t^p} \tag{37}$$

$$v_t^p = \int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} di \tag{38}$$

$$P_t^{1-\epsilon} = \int_0^1 P_t(i)^{1-\epsilon} di \tag{39}$$

$$P_t(i) = \frac{\epsilon}{\epsilon - 1} \frac{X_{1,t}}{X_{2,t}} \tag{40}$$

$$X_{1,t} = \lambda_t mc_t P_t^{\epsilon} Y_t + \beta \xi_p \mathbb{E}_t X_{1,t+1} \tag{41}$$

$$X_{2,t} = \lambda_t P_t^{\epsilon-1} Y_t + \beta \xi_p \mathbb{E}_t X_{2,t+1} \tag{42}$$

$$\left(\frac{R_t}{R} \right) = \left(\frac{R_{t-1}}{R} \right)^{\rho_{R,1}} \left(\frac{R_{t-2}}{R} \right)^{\rho_{R,2}} \left[\left(\frac{\pi_t}{\pi_t^{\star}} \right)^{\alpha_\pi} \left(X_t \right)^{\alpha_x} \left(\frac{Y_t}{Y_{t-1}} \right)^{\alpha_{\Delta y}} \right]^{1-\rho_{R,1}-\rho_{R,2}} v_t^r \tag{43}$$

$$X_t = \frac{Y_t}{Y_t^n} \tag{44}$$

$$v \left(\frac{Y_t^n}{A_t} \right)^{1+\eta} = \left(\frac{\epsilon - 1}{\epsilon} \right) + vh \left(\frac{Y_t^n}{A_t} \right) \left(\frac{Y_{t-1}^n}{A_t} \right) \tag{45}$$

$$\log g_{A,t} = (1-\rho_z)\log g_A + \rho_z \log g_{A,t-1} + \epsilon_t^z \tag{46}$$
$$\log b_t = (1-\rho_b)\log b + \rho_b \log b_{t-1} + \epsilon_t^b \tag{47}$$
$$\log v_t^r = \rho_r \log v_{t-1}^r + \epsilon_t^r \tag{48}$$
$$\log \pi_t^\star = (1-\rho_\pi)\pi + \rho_\pi \log \pi_{t-1}^\star + \epsilon_t^\pi \tag{49}$$

C Log-linearized model

After detrending the model by removing trend growth, the log-linearized model can be characterized by 11 endogenous variables and 11 equations which are described below.

$$\begin{aligned} \tilde{y}_t =& \frac{h}{h+g_A}\left(\tilde{y}_{t-1} - \tilde{g}_{A,t}\right) + \frac{g_A}{h+g_A}\mathbb{E}_t\left(\tilde{y}_{t+1} + \tilde{g}_{A,t+1}\right) \\ & - \frac{g_A - h}{h+g_A}\left(\tilde{r}_t - \mathbb{E}_t\tilde{\pi}_{t+1} - \tilde{b}_t + \mathbb{E}_t\tilde{b}_{t+1}\right) \end{aligned} \tag{50}$$

$$\begin{aligned} \tilde{\pi}_t =& \beta[1+\epsilon(1-\xi_p\pi^{\epsilon-1})(\pi-1)]\mathbb{E}_t\tilde{\pi}_{t+1} + \beta(1-\xi_p\pi^{\epsilon-1})(\pi-1)\mathbb{E}_t\tilde{x}_{1,t+1} \\ & + \left(\frac{(1-\xi_p\pi^{\epsilon-1})(1-\beta\xi_p\pi^{\epsilon})}{\xi_p\pi^{\epsilon-1}}\right)((1+\eta)\tilde{y}_t + \eta\tilde{v}_t^p) + \beta(1-\pi)(1-\xi_p\pi^{\epsilon-1})\tilde{b}_t \\ & + \left(\frac{(1-\beta\xi_p\pi^{\epsilon-1})(1-\xi_p\pi^{\epsilon-1})}{\xi_p\pi^{\epsilon-1}}\right)\left(\frac{h}{g_A - h}\right)(\tilde{y}_t - \tilde{y}_{t-1} + \tilde{g}_{A,t}) \end{aligned} \tag{51}$$

$$\tilde{x}_{1,t} = (1-\beta\xi_p\pi^{\epsilon})(\tilde{b}_t + (1+\eta)\tilde{y}_t + \eta\tilde{v}_t^p) + \beta\xi_p\pi^{\epsilon}\mathbb{E}_t[\tilde{x}_{1,t+1} + \epsilon\tilde{\pi}_{t+1}] \tag{52}$$

$$\tilde{v}_t^p = \frac{\epsilon\xi_p\pi^{\epsilon-1}(\pi-1)}{1-\xi_p\pi^{\epsilon-1}}\tilde{\pi}_t + \pi^{\epsilon}\xi_p\tilde{v}_{t-1}^p \tag{53}$$

$$\tilde{y}_t^n = \frac{h}{(1+\eta)g_A - h\eta}\left(\tilde{y}_{t-1}^n - \tilde{g}_{A,t}\right) \tag{54}$$

$$\begin{aligned} \tilde{r}_t =& \rho_{R,1}\tilde{r}_{t-1} + \rho_{R,2}\tilde{r}_{t-2} \\ & + (1-\rho_{R,1}-\rho_{R,2})\left(\alpha_\pi(\tilde{\pi}_t - \tilde{\pi}_t^\star) + \alpha_x\tilde{x}_t + \alpha_{\Delta y}(\tilde{y}_t - \tilde{y}_{t-1} + \tilde{g}_{A,t})\right) + \tilde{v}_t^r \end{aligned} \tag{55}$$

$$\hat{x}_t = \hat{y}_t - \hat{y}_t^n \tag{56}$$

$$\tilde{g}_{A,t} = \rho_z \tilde{g}_{A,t-1} + \epsilon_t^z \tag{57}$$

$$\tilde{b}_t = \rho_b \tilde{b}_{t-1} + \epsilon_t^b \tag{58}$$

$$\tilde{v}_t^r = \rho_r \tilde{v}_{t-1}^r + \epsilon_t^r \tag{59}$$

$$\tilde{\pi}_t^\star = \rho_\pi \tilde{\pi}_{t-1}^\star + \epsilon_t^\pi \tag{60}$$

Table 1: Prior distributions

Parameter	Domain	Density	Para(1)	Para(2)
h	[0,1)	Beta	0.7	0.1
ξ_p	[0,1)	Beta	0.66	0.1
α_π	$\mathbb{R}^+$	Normal	1.5	0.3
α_x	$\mathbb{R}^+$	Normal	0.125	0.05
$\alpha_{\Delta y}$	$\mathbb{R}^+$	Normal	0.125	0.05
$\rho_{R,1}$	$\mathbb{R}^+$	Normal	1.1	0.25
$\rho_{R,2}$	$\mathbb{R}$	Normal	0	0.25
$\bar{A}$	$\mathbb{R}$	Normal	0.4	0.2
$\bar{\pi}$	$\mathbb{R}$	Normal	0.8	0.75
$\bar{r}$	$\mathbb{R}^+$	Normal	1.2	0.4
ρ_b	[0,1)	Beta	0.5	0.2
ρ_z	[0,1)	Beta	0.5	0.2
ρ_r	[0,1)	Beta	0.5	0.2
ρ_π	[0,1)	Beta	0.5	0.2
σ_b	$\mathbb{R}^+$	InvGamma	0.5	4
σ_z	$\mathbb{R}^+$	InvGamma	0.5	4
σ_r	$\mathbb{R}^+$	InvGamma	0.5	4
σ_π	$\mathbb{R}^+$	InvGamma	0.5	4
σ_s	$\mathbb{R}^+$	InvGamma	0.5	4
α_{BN}	[0.5,1.5]	Uniform	0.5	1.5
$\rho_{b,s}$	[-1,1]	Uniform	-1	1
$\rho_{z,s}$	[-1,1]	Uniform	-1	1
$\rho_{r,s}$	[-1,1]	Uniform	-1	1
$\rho_{\pi,s}$	[-1,1]	Uniform	-1	1

Notes: For the beta and normal densities, Para(1) and Para(2) refer to the means and standard deviations of the prior. For the Uniform densities, Para(1) and Para(2) refer to the lower and upper bounds. For the Inverse Gamma distribution, Para(1) and Para(2) refer to s and v where $p_{IG}(\sigma|v,s) \propto \sigma^{-v-1}e^{-vs^2/2\sigma^2}$.

Table 2: Estimation of simulated data

	Indeterminacy		Determinacy	
	True	**75 observations**	**True**	**75 observations**
h	0.85	0.771 [0.707,0.838]	0.85	0.820 [0.778,0.863]
ξ_p	0.75	0.719 [0.627,0.819]	0.75	0.735 [0.684,0.781]
α_π	0.8	0.943 [0.767,1.121]	1.9	1.702 [1.462,1.945]
α_x	0.12	0.144 [0.065,0.218]	0.12	0.115 [0.035,0.184]
$\alpha_{\Delta y}$	0.18	0.127 [0.045,0.202]	0.18	0.121 [0.050,0.198]
$\rho_{R,1}$	0.90	0.902 [0.690,1.103]	0.90	0.942 [0.793,1.094]
$\rho_{R,2}$	-0.25	-0.353 [-0.488,-0.218]	-0.25	-0.376 [-0.491,-0.258]
$\bar{A}$	0.50	0.564 [0.388,0.762]	0.50	0.430 [0.225,0.624]
$\bar{\pi}$	1.00	0.740 [0.533,0.955]	1.00	0.989 [0.846,1.124]
$\bar{r}$	1.40	1.015 [0.790,1.264]	1.40	1.264 [0.990,1.539]
ρ_b	0.80	0.552 [0.239,0.862]	0.80	0.753 [0.656,0.870]
ρ_z	0.25	0.399 [0.165,0.647]	0.25	0.247 [0.100,0.388]
ρ_r	0.50	0.551 [0.236,0.623]	0.50	0.514 [0.389,0.652]
ρ_π	0.99	0.887 [0.805,0.980]	0.99	0.953 [0.920,0.988]
σ_b	1.00	0.522 [0.284,0.764]	1.00	0.829 [0.584,1.046]
σ_z	1.20	0.760 [0.474,1.090]	1.20	1.091 [0.856,1.303]
σ_r	0.30	0.311 [0.260,0.358]	0.30	0.358 [0.288,0.425]
σ_π	0.05	0.126 [0.109,0.142]	0.05	0.125 [0.109,0.141]
σ_s	0.50	0.564 [0.484,0.650]	0.50	0.579 [0.279,0.887]
α_{BN}	0.50	0.762 [0.557,0.997]	1.50	1.240 [1.003,1.443]
$\rho_{b,s}$	0.00	-0.079 [-0.811,0.613]	0.00	-0.003 [-0.647,0.667]
$\rho_{z,s}$	0.00	-0.146 [-0.439,0.133]	0.00	-0.070 [-0.719,0.583]
$\rho_{r,s}$	0.00	-0.110 [-0.490,0.285]	0.00	-0.004 [-0.688, 0.651]
$\rho_{\pi,s}$	0.00	-0.125 [-0.465,0.269]	0.00	0.042 [-0.600, 0.718]
Prob(det)		0.0000		1.0000

Notes: In the estimates columns, the numbers in brackets are 90% HPDI intervals. The priors used in the estimation of the simulated data are the same as the priors listed in Table 1.

Table 3: AR(2) Policy Rules 1964Q1:1979Q2

	Persistent policy shocks		**White noise policy shocks**	
	Mixed rule	**Growth rule**	**Mixed rule**	**Growth rule**
h	0.500	0.469	0.542	0.472
	[0.370,0.641]	[0.358,0.573]	[0.419,0.661]	[0.359,0.585]
ξ_p	0.626	0.622	0.696	0.577
	[0.521,0.711]	[0.540,0.713]	[0.526,0.852]	[0.469,0.683]
α_π	1.999	2.125	1.399	2.092
	[0.790,2.596]	[1.808,2.476]	[0.734,2.254]	[1.757,2.405]
α_x	0.119	—	0.158	—
	[0.044,0.195]	[—,—]	[0.077,0.236]	[—,—]
$\alpha_{\Delta y}$	0.148	0.135	0.136	0.130
	[0.079,0.223]	[0.064,0.208]	[0.066,0.209]	[0.053,0.202]
$\rho_{R,1}$	0.834	0.807	1.051	0.971
	[0.616,1.069]	[0.600,1.026]	[0.814,1.257]	[0.782,1.169]
$\rho_{R,2}$	-0.317	-0.338	-0.313	-0.358
	[-0.502,-0.130]	[-0.525,-0.130]	[-0.473,-0.141]	[-0.526,-0.163]
$\bar{A}$	0.442	0.451	0.417	0.445
	[0.195,0.694]	[0.203,0.692]	[0.190,0.663]	[0.189,0.680]
$\bar{\pi}$	1.068	1.043	1.148	1.049
	[0.861,1.277]	[0.832,1.259]	[0.921,1.390]	[0.851,1.253]
$\bar{r}$	1.359	1.324	1.453	1.339
	[1.117,1.596]	[1.099,1.542]	[1.260,1.654]	[1.165,1.502]
ρ_b	0.728	0.793	0.561	0.799
	[0.424,0.896]	[0.710,0.883]	[0.243,0.895]	[0.724,0.895]
ρ_z	0.185	0.160	0.402	0.171
	[0.052,0.272]	[0.062,0.249]	[0.073,0.766]	[0.077,0.265]
ρ_r	0.380	0.373	—	—
	[0.204,0.545]	[0.220,0.528]	[—,—]	[—,—]
ρ_π	0.916	0.932	0.930	0.933
	[0.865,0.971]	[0.887,0.983]	[0.890,0.974]	[0.888,0.977]
σ_b	1.226	1.310	1.303	1.473
	[0.680,1.834]	[0.922,1.695]	[0.380,1.984]	[1.070,1.924]
σ_z	1.596	1.520	1.257	1.521
	[1.083,2.177]	[1.212,1.833]	[0.508,1.807]	[1.162,1.833]
σ_r	0.366	0.369	0.269	0.314
	[0.245,0.480]	[0.273,0.474]	[0.215,0.330]	[0.249,0.381]
σ_π	0.151	0.148	0.144	0.147
	[0.127,0.173]	[0.126,0.171]	[0.122,0.164]	[0.124,0.167]
σ_s	0.513	0.571	0.458	0.604
	[0.288,0.741]	[0.275,0.865]	[0.343,0.572]	[0.280,0.926]
α_{BN}	1.195	1.262	0.936	1.251
	[0.804,1.500]	[1.052,1.495]	[0.514,1.303]	[1.028,1.470]
$\rho_{b,s}$	-0.005	-0.027	0.077	-0.014
	[-0.679,0.676]	[-0.697,0.621]	[-0.381,0.598]	[-0.695,0.633]
$\rho_{z,s}$	-0.006	0.014	-0.126	-0.011
	[-0.594,0.601]	[-0.645,0.684]	[-0.478,0.270]	[-0.719,0.622]
$\rho_{r,s}$	0.016	-0.040	-0.088	-0.006
	[-0.580,0.672]	[-0.715,0.610]	[-0.627,0.269]	[-0.633,0.685]
$\rho_{\pi,s}$	0.130	-0.013	0.402	0.007
	[-0.471,0.878]	[-0.698,0.655]	[-0.454,0.897]	[-0.697,0.651]
log p(X^T)	-93.1928	-91.7187	-95.5699	-95.5688
Prob(det)	0.8489	0.9928	0.3271	1.0000

Table 4: AR(2) Policy Rules 1983Q1:2005Q1

	Persistent policy shocks		**White noise policy shocks**	
	Mixed rule	**Growth rule**	**Mixed rule**	**Growth rule**
h	0.635	0.659	0.784	0.710
	[0.546,0.739]	[0.571,0.749]	[0.682,0.880]	[0.609,0.809]
ξ_p	0.806	0.817	0.863	0.803
	[0.769,0.843]	[0.785,0.848]	[0.835,0.902]	[0.754,0.851]
α_π	2.414	2.372	1.241	1.984
	[2.049,2.750]	[2.018,2.746]	[0.835,1.607]	[1.537,2.420]
α_x	0.085	—	0.144	—
	[0.006,0.150]	[—,—]	[0.105,0.191]	[—,—]
$\alpha_{\Delta y}$	0.166	0.173	0.158	0.203
	[0.099,0.232]	[0.106,0.240]	[0.092,0.220]	[0.130,0.281]
$\rho_{R,1}$	0.687	0.683	1.275	1.262
	[0.525,0.860]	[0.512,0.836]	[1.148,1.427]	[1.112,1.435]
$\rho_{R,2}$	-0.254	-0.263	-0.353	-0.416
	[-0.418,-0.099]	[-0.423,-0.105]	[-0.521,-0.208]	[-0.582,-0.268]
$\bar{A}$	0.525	0.482	0.269	0.480
	[0.349,0.708]	[0.283,0.677]	[-0.004,0.498]	[0.278,0.687]
$\bar{\pi}$	0.784	0.855	0.801	0.816
	[0.617,0.948]	[0.682,1.032]	[0.695,0.923]	[0.676,0.947]
$\bar{r}$	1.508	1.568	1.283	1.476
	[1.313,1.732]	[1.305,1.812]	[1.067,1.490]	[1.284,1.661]
ρ_b	0.848	0.838	0.437	0.794
	[0.784,0.916]	[0.775,0.906]	[0.270,0.567]	[0.686,0.894]
ρ_z	0.177	0.186	0.711	0.229
	[0.047,0.300]	[0.044,0.343]	[0.458,0.962]	[0.048,0.417]
ρ_r	0.767	0.771	—	—
	[0.703,0.834]	[0.708,0.835]	[—,—]	[—,—]
ρ_π	0.930	0.956	0.958	0.920
	[0.893,0.979]	[0.922,0.990]	[0.930,0.992]	[0.870,0.970]
σ_b	1.227	1.189	1.603	1.592
	[0.875,1.533]	[0.876,1.520]	[1.029,2.186]	[1.132,2.015]
σ_z	1.112	1.174	0.863	1.264
	[0.879,1.355]	[0.929,1.432]	[0.274,1.420]	[0.930,1.586]
σ_r	0.256	0.246	0.163	0.185
	[0.203,0.306]	[0.195,0.298]	[0.137,0.185]	[0.159,0.214]
σ_π	0.113	0.111	0.134	0.116
	[0.100,0.126]	[0.097,0.124]	[0.118,0.154]	[0.100,0.129]
σ_s	0.550	0.695	0.308	0.563
	[0.271,0.818]	[0.283,1.136]	[0.191,0.379]	[0.288,0.856]
α_{BN}	1.281	1.209	0.750	1.230
	[1.081,1.500]	[1.000,1.411]	[0.517,0.899]	[1.001,1.436]
$\rho_{b,s}$	0.126	-0.132	-0.055	-0.018
	[-0.485,0.746]	[-0.694,0.503]	[-0.419,0.352]	[-0.671,0.665]
$\rho_{z,s}$	-0.156	0.044	-0.027	0.089
	[-0.787,0.457]	[-0.558,0.753]	[-0.306,0.241]	[-0.578,0.713]
$\rho_{r,s}$	0.083	0.011	0.138	-0.064
	[-0.538,0.718]	[-0.670,0.671]	[-0.078,0.372]	[-0.737,0.559]
$\rho_{\pi,s}$	0.065	-0.036	0.743	0.056
	[-0.604,0.690]	[-0.669,0.632]	[0.545,0.950]	[-0.601,0.702]
log p(X^T)	61.5134	66.3211	36.2191	38.5321
Prob(det)	1.000	1.000	0.0626	1.0000

Table 5: Additional Results, 1964Q1:1979Q2

	Panel A: AR(1) Policy Rules			
	Persistent policy shocks		**White noise policy shocks**	
	Mixed rule	**Growth rule**	**Mixed rule**	**Growth rule**
log p(X^T)	-94.0730	-93.6336	-99.0035	-98.1708
Prob(det)	0.0101	1.000	0.7755	0.9999

	Panel B: AR(2) Policy Rules with No Target Observable			
	Standard priors		**Persistent priors**	
	Mixed rule	**Growth rule**	**Mixed rule**	**Growth rule**
ρ_π	0.485	0.503	0.948	0.950
log p(X^T)	-130.4705	-132.0124	-125.5061	-125.4843
Prob(det)	0.5089	0.4243	0.9929	0.9949

	Panel C: AR(1) Policy Rules with Fixed Inflation Target			
	Persistent policy shocks		**White noise policy shocks**	
	Mixed rule	**Growth rule**	**Mixed rule**	**Growth rule**
log p(X^T)	-127.9917	-130.2562	-132.7940	-134.6669
Prob(det)	0.0091	0.4786	0.0004	0.0004

Notes: Panel A reports estimates for the MOT and OGT with AR(1) smoothing and a time-varying inflation target, with the inflation target observable. Panel B reports estimates for MOT and OGT with AR(2) smoothing, persistent policy shocks, a time-varying inflation target, but with no observable for the inflation target. Panel C reports estimates for MOT and OGT with AR(1) smoothing, persistent/white noise policy shocks, and a fixed inflation target.

Table 6: Data and Model Implied Volatility

	Data		**AR(2) OGT Model**		**AR(1) MOT Model**	
	1964-1979	1983-2005	1964-1979	1983-2005	1964-1979	1983-2005
$\sigma(\Delta Y)$	1.02	0.56	1.19	0.84	1.71	0.78
			[0.93,1.46]	[0.59,1.09]	[0.86,2.86]	[0.58,1.01]
$\sigma(\pi_t)$	0.61	0.22	0.74	0.61	1.19	0.55
			[0.47,1.04]	[0.35,0.78]	[0.72,1.71]	[0.33,0.81]
$\sigma(R_t)$	0.55	0.64	0.76	0.55	1.18	0.49
			[0.50,1.06]	[0.34,0.78]	[0.67,1.77]	[0.31,0.70]
$\sigma(\pi_t^\star)$	0.49	0.22	0.30	0.25	0.21	0.22
			[0.17,0.45]	[0.14,0.37]	[0.11,0.33]	[0.13,0.32]

Notes: Numbers in square brackets are 90% confidence intervals based on posterior parameter and small sample uncertainty. We compute these by drawing repeatedly from the posterior densities, simulating the model for 80 time periods, and computing standard deviations of the observables. The model implied volatilities are the mean volatilities over 20,000 draws.

Figure 1: Determinacy regions with MOT

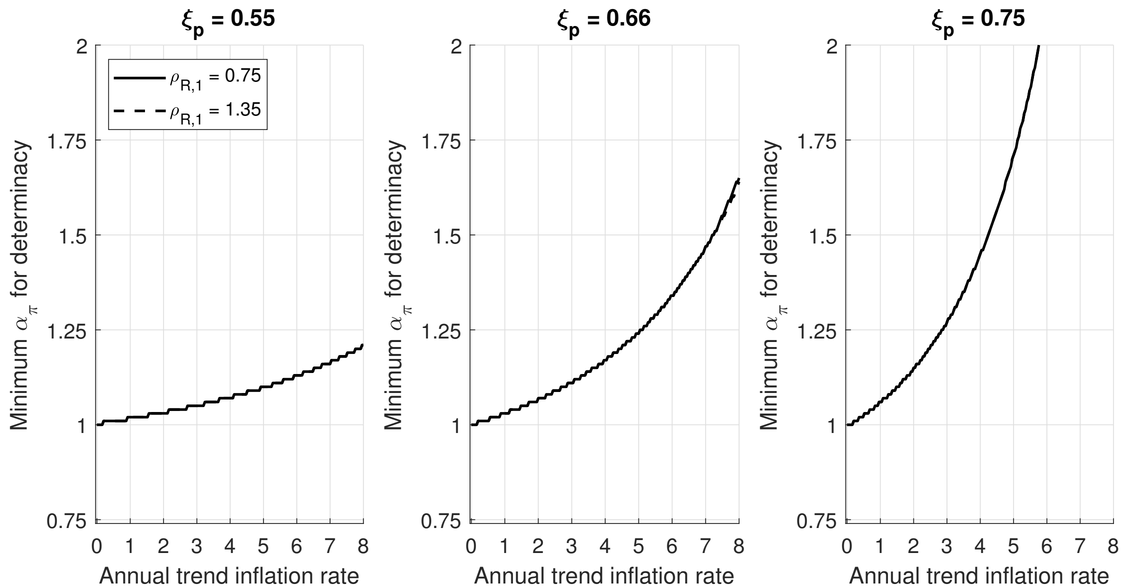

Figure 2: Determinacy regions with OGT