Has the US Business Cycle Become More Asymmetric?*

Joshua Brault†

May 2019

Abstract

This paper documents that the US business cycle has been characterized by increasingly asymmetric fluctuations since the onset of the Great Moderation. We distinguish between two different forms of asymmetry, steepness and deepness. Evidence suggests that US GDP, consumption and investment have been characterized by increasingly deep fluctuations, however the increases in the steepness of U.S. GDP is driven largely by changes to fluctuations in investment. We contrast this evidence with DSGE models featuring occasionally binding collateral constraints and highlight these models primarily produce deepness.

*I am thankful to Ba Chu, Dana Galizia, Chris Gunn, Lilia Karnizova, Lynda Khalaf, Hashmat Khan, Lutz Killian, Minjoon Lee, Markus Poschke and Abdallah Zalghout for comments and insightful discussions. Any remaining errors are my own.

†Department of Economics, D886 Loeb, 1125 Colonel By Drive, Carleton University, Ottawa, Canada. *E-mail:* joshua.brault@carleton.ca. Tel: +1.613.520.2600 (ext 3057).

1 Introduction

The observation that the US business cycle is characterized by asymmetric fluctuations dates at least as far back as Mitchell (1927, pg. 333), who noted that "business contractions appear to be briefer and more violent than business expansions". Recent empirical evidence has documented, both in the US and internationally, that the business cycle has become increasingly asymmetric (e.g., Popov (2014); Jordà et al. (2016); Jensen et al. (2017); Garin et al. (2018)). However, the asymmetry of focus has typically been on the negative skewness in the growth rate of real GDP.

The contribution of this paper is twofold. First, we examine the evidence for changes in the asymmetry of the US business cycle since the mid-1980s through the lens of two different forms of asymmetry, deepness and steepness (Sichel 1993). Deep cycles describe cyclical behavior where troughs are further from trend than peaks are above. Steep cycles are characterized by cyclical behavior where the speed of contraction is larger in absolute value than the speed of expansion. The sample split we consider has been widely studied in the literature in the context of declining macroeconomic volatility and changes in the cyclicality of macroeconomic variables (e.g., McConnell and Perez-Quiros (2000); Brault and Khan (2018)). This paper defines the business cycle using the Hamilton (2017) filter and quarter over quarter growth rates. While a large amount of research has examined US macroeconomic time series for these two types of asymmetry[1], the changes in asymmetry has not been examined from this perspective. A key distinction this paper makes is that in addition to examining US real GDP as the measure of the business cycle, we also investigate asymmetry in the two largest components of GDP, consumption (aggregate, durable and non-durable/services) and investment (aggregate private, residential, and non-residential). This distinction turns out to be important in explaining asymmetry in GDP as consumption and investment components feature different magnitudes and types of asymmetry, suggesting relevant and empirically important macroeconomic channels not captured by examining GDP alone.

Secondly, we highlight that models with occasionally binding collateral constraints produce a particular form of asymmetry, deepness. While other mechanisms are capable of producing asymmetry over the business cycle, the focus on the collateral channel in this paper arises due to recent attempts to motivate this channel through skewness in the growth rate of real GDP (for

[1] See, for example, Sichel (1993), Verbrugge (1997), Razzak (2001), Hansen and Prescott (2005), Atolia et al. (2018), Ferraro (2018).

example, Jensen et al. (2017)). To highlight that deepness is the primary asymmetry generated by occasionally binding collateral constraint models, we first present a simple partial equilibrium model where housuing serves as collateral for borrowing in the presence of exogenous house price shocks. To confirm these findings translate into general equilibrium, we present evidence from a recent DSGE model via Guerrieri and Iacoviello (2017).

Results indicate that US GDP, consumption, and investment have been characterized by increasingly deep cycles since the onset of the Great Moderation. A robust feature of this pattern is that all series considered show deepness coefficients consistent with increasingly asymmetric fluctuations even if the Great Recession is excluded from the post-Great Moderation sample. The evidence for increasing steepness is not uniform across all variables. Consistent with the previous literature, we find that growth rate of real GDP has become much more negatively skewed (e.g., Jensen et al. (2017); Garin et al. (2018)). However, a key insight provided in this paper is that the increasing steepness of GDP is driven largely by investment. Steepness in consumption has remained relatively unchanged since the onset of the Great Moderation. In contrast, both residential and non-residential investment have experienced large increases in the steepness of their cycles, even if the Great Recession is excluded.

This paper proceeds as follows: In Section 2 we describe deepness and steepness in further detail and how to test for changes in these types of asymmetry. In Section 3 we report the empirical evidence for changes in deepness and steepness in output, consumption components, and investment components. In Section 4 we describe a partial equilibrium model with an occasionally binding constraint alongside results from a modern DSGE model. Section 5 concludes.

2 Deepness and Steepness

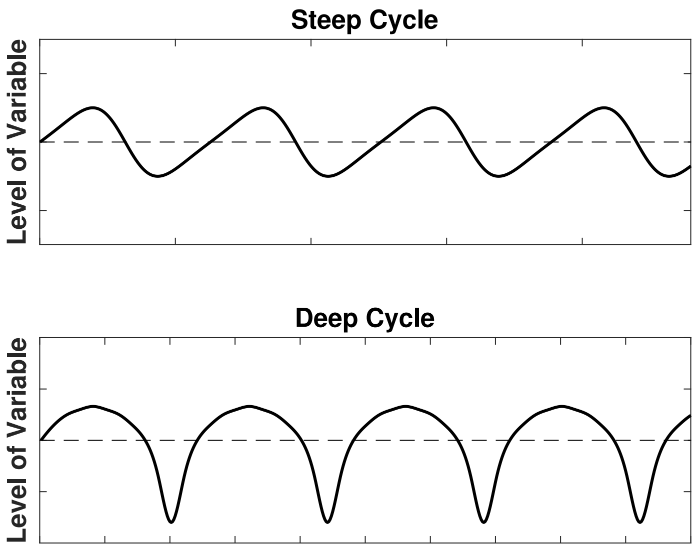
Deepness refers to cyclical behavior where busts are further from trend than booms are above.[2,3] Steepness describes cyclical behavior where the speed of contractions is larger in absolute magnitude than the speed of expansions. Under these definitions, deepness implies that the probability

[2]Alternatively, deepness could be interpreted as the average deviation above trend being smaller than the average deviation below trend as in Hansen and Prescott (2005).

[3]In this sense I am using boom and busts as described by McKay and Reis (2006). Expansions and contractions characterize the behavior of a macroeconomic time series from trough to peak and peak to trough. Booms and busts refer periods where the time series is above or below trend. A boom then includes the latter part of the expansion and beginning of the contraction.

density function (PDF) of the cyclical component will exhibit negative skewness while steepness implies that the PDF of the growth rate will exhibit negative skewness. These distinct forms of asymmetry can occur separately or together. Figure 1 plots hypothetical deep and steep cycles,

Figure 1: Steep and Deep Business Cycles (Sichel 1993)



While the focus thus far has been on negative skewness in the distribution of the cyclical component and of the growth rates, deepness and steepness also have several other predictions for the behavior of macroeconomic series over the business cycle. Deepness implies that the average of the cyclical component above trend is less than the average of the cyclical component below trend. In addition, the duration of booms relative to busts is increasing in the degree of deepness. Steepness implies that the average growth rate during expansions is smaller in absolute magnitude than growth rates during contractions. In addition, the duration of expansions relative to contractions is increasing in the degree of steepness. These informal measures of asymmetry are used to support evidence of changes in deepness and steepness.

2.1 Detrending

As discussed by Sichel (1993, pg. 228), testing for asymmetry requires the following to be satisfied: First, each series must be rendered stationary, since by definition most macroeconomic time series are upward trending. Second, the detrending method must extract the exact component to be used in the test. For deepness this is the cyclical component in deviation from trend, and steepness, the first-difference (i.e., the growth rate since all variables are log transformed). Finally, the detrending method must not induce any additional asymmetry into the series.

To obtain the cyclical component in deviation from trend, I use the filter suggested by Hamilton (2017). The filter amounts to obtaining obtaining forecast residuals from a univariate regression h periods ahead using the p most recent realizations as explanatory variables. I use Hamilton's suggested parametric specification for detrending quarterly data which sets $h = 8$ and $p = 4$,

$$\hat{v}_{t+h} = y_{t+h} - \hat{\beta}_0 + \hat{\beta}_1 y_t + \hat{\beta}_2 y_{t-1} + \hat{\beta}_3 y_{t-2} + \hat{\beta}_4 y_{t-3}$$

Since the variables are log transformed, the cyclical components from the filter have the interpretation of being the percentage deviation from trend. Testing for steepness requires testing on the rate of change in each series. I employ the first-difference filter and since the variables are log transformed this amounts to hypothesis testing on the growth rates of variables.

2.2 Coefficient of Skewness Test

To test for deepness and steepness this paper uses a coefficient of skewness test applied to cyclical components and growth rates as in Sichel (1993). Since the particular form of asymmetry this paper is interested in is deep recessions and steep downturns, the null hypothesis is that the reference series contains no deepness or steepness, and the alternative is that there is evidence of deepness or steepness. This leads to the following one-tailed hypothesis test,

$$H_0 : \text{Deepness/Steepness} \geq 0$$
$$H_A : \text{Deepness/Steepness} < 0$$

and using the cyclical components obtained from the Hamilton filter, the deepness test statistic is computed using the familiar coefficient of skewness formula,

$$\hat{\tau} = \frac{\mathrm{E}[(c-\mu_c)^3]}{\mathrm{E}[(c-\mu_c)^2]^{3/2}} = \frac{\mu_3}{\sigma^3}$$

where c and μ_c represent the cyclical component and mean of the cyclical component for each reference series, respectively.[4] The coefficient of skewness will be negative if the average deviation in busts is further from trend than the average deviation is above in booms. Analogously, the test for steepness is computed using the coefficient of skewness, except the cyclical components, c, obtained through the Hamilton filter are replaced with growth rates obtained after taking first-difference of the log of the reference series. The test for steepness will be negative if the growth rates during contractions are larger in absolute magnitude than growth rates experienced during expansions.

The coefficient of skewness test follows an asymptotic normal distribution only when data are assumed to be independent and identically distributed. However, both components in growth rates and cyclical components obtained through the Hamilton filter contain an unknown form of serial correlation.[5]. To compute standard errors this paper uses a correction suggested by Bai and Ng (2005, see *Corollary 1*, pg. 50).[6]

Finally, after computing sample skewness for the subsamples, we test whether sample skewness coefficients are statistically different since the onset of the Great Moderation (defined as post-1984). Since sample skewness coefficients for deepness and steepness are negative, a more asymmetric sample skewness in the post-1984 period would suggest the appropriate hypotheses,

$$H_0 : \hat{\tau}_1 - \hat{\tau}_2 \leq 0$$
$$H_A : \hat{\tau}_1 - \hat{\tau}_2 > 0$$

which I test using a test of the following form,

[4]Since the Hamilton (2017) filter contains a constant, μ_c is approximately zero for all the series.

[5]Ljung-Box test statistics for cyclical components and growth rates are reported in Table 7. The tests allow for serial correlation up to order 10. The null hypothesis that the data are i.i.d. is rejected for all variables in levels and growth rates at the 1% level

[6]For a further description of the Bai and Ng (2005) approach to calculating standard errors for sample skewness in the presence of serial correlation, see the appendix.

$$Z = \frac{(\hat{\tau}_1 - \hat{\tau}_2) - (\mu_1 - \mu_2)}{\sqrt{\sigma^2_{\tau_1} + \sigma^2_{\tau_2}}}$$

To compute P-values for the test we use a circular block bootstrap with 9,999 replications. The block bootstrap routine resamples blocks of observations from the two samples (pre- and post Great Moderation) to approximate empirical distributions which we use to test the hypothesis. Block length is chosen according to the rule-of-thumb $B = T^{1/3}$.[7] These results are discussed in Section 3.

3 Results

The hypotheses discussed above are applied to US GDP, consumption, and investment.[8] I present results that distinguish between components of consumption (aggregate, durable, and non-durable/services) and components of investment (gross private, non-residential, and residential). The data span 1947:I-2018:II, but since the Hamilton filter loses 11 observations from the beginning of the sample, all tests are performed on data from 1949:4-2018:2. I consider three subsamples in the data: 1949:4-1983:4 (pre-1984), 1984:1-2007:4 (Great Moderation), 1984:1-2018:2 (post-1984). The purpose of the Great Moderation sample is to assess the extent to which asymmetry results are driven by the Great Recession. However, statistical tests on the difference in sample skewness are only performed on the pre-1984 and post-1984 samples.

Before presenting sample skewness coefficients, I present some informal measures of business cycle asymmetry relating to average booms, busts, expansions, and contractions that were discussed in relation to deepness and steepness in Section 2.[9] Table 1 reports the average boom, bust, expansion growth rate, and contraction growth rate. Expansion and contraction growth

[7]Results are generally insensitive to the choice of block size. Alternative rules of thumb included $T^{1/2}$ and $T^{1/4}$. The conclusions reached in the baseline case remain unchanged.

[8]Data are obtained from the Federal Reserve Bank of St. Louis Economic Database (FRED). All nominal variables are converted to real measures using their corresponding price deflators. FRED codes are provided in Table 6 in the Appendix.

[9]The emphasis on average booms, busts, expansions, and contractions as opposed to durations of these phases is due to the splitting of the sample. For example, the decision to include a boom spanning the split in the two samples into the pre-1984 or post-1984 sample is not obvious. In addition this choice is quantitatively important for the results. Instead I focus on averages of the phases which does not require this decision.

rates reported are demeaned to highlight the fundamental asymmetry. The Hamilton (2017) filtered data are by definition approximately mean zero because the filter includes a constant in the regression.

Table 1: Average Booms, Busts, Expansions and Contractions

Variable	Sample	Boom	Bust	Expansion	Contraction
GDP	1949.3-1983.4	2.795	-3.579	0.315	-1.647
	1984.1-2018.2	2.027	-2.196	0.083	-1.351
Consumption	1949.3-1983.4	2.223	-2.269	0.447	-0.802
	1984.1-2018.2	1.802	-2.382	0.086	-0.903
Durable Consumption	1949.3-1983.4	7.514	-10.305	1.984	-3.563
	1984.1-2018.2	5.461	-9.069	0.390	-4.099
Non-durable/Services	1949.3-1983.4	1.568	-1.670	0	—[a]
	1984.1-2018.2	1.303	-1.795	0.042	-0.925
Gross Private Investment	1949.3-1983.4	9.566	-11.701	2.793	-4.565
	1984.1-2018.2	9.928	-10.697	1.213	-2.869
Non-residential Inv.	1949.3-1983.4	7.610	-7.769	1.111	-2.791
	1984.1-2018.2	7.197	-8.672	0.678	-2.662
Residential Inv.	1949.3-1983.4	13.688	-18.639	3.788	-4.720
	1984.1-2018.2	13.501	-18.145	1.566	-3.346

Note: Booms are defined as periods above trend defined from the Hamilton filter and busts are defined as periods below trend. To identify expansions and contractions I use the BBQ algorithm defined by Harding and Pagan (2002) which expands on the business cycle turning point algorithm of Bry and Boschan (1971). Average expansion and contraction growth rates are demeaned.
[a] The BBQ algorithm does not identify a contraction in consumption of non-durables and services from 1949:4-1983:4.

Several common patterns emerge across the variables presented. First, aside from gross private investment, all variables experience a fall in the average boom in the post-1984 data. In contrast the average bust across variables is much more mixed. Turning to expansions and contractions, a similar pattern emerges; all variables experience a fall in the average growth rate during expansions in the post-1984 data.[10] While contraction growth rates have fallen across most variables, these changes are relatively small in comparison to expansions. This evidence

[10]This statement does not hold for non-durable consumption. However this result should be interpreted with caution. As the footnote in Table 1 mentions, the BBQ algorithm does not identify a contraction in non-durable consumption during the pre-1984 data and as such the demeaned expansion growth rate is zero by definition.

is consistent with the literature which suggests that changing business cycle fluctuations are driven largely by changes in the expansion phase of the business cycle (see, e.g., Fatàs and Mihov (2013)). This evidence is suggestive of changing business cycle asymmetry consistent with both deepness and steepness. Booms and expansions have become less apparent while busts and contractions have remained relatively unchanged. Turning to more formal statistical measures of asymmetry, Table 2 presents sample skewness coefficients across the subsamples.

Sample skewness coefficients for deepness provide two key insights. First, there is strong statistical evidence that fluctuations in GDP, consumption, and investment are characterized by negative skewness in the distribution of the cyclical component in the post-1984 data. This result contrasts the pre-1984 data where there is little evidence for deepness, consistent with the existing literature (e.g., Sichel (1993)). This result suggests that fluctuations in these variables feature deep troughs and relatively modest booms since the mid-1980s. The second insight is that the sample skewness coefficients for all variables have become increasingly asymmetric across the subsamples. A robust feature of this pattern is that deepness is increasing even if the Great Recession is excluded from the post-1984 sample. Not surprisingly, the largest increase in deepness arises in the residential investment sector, but all other variables also show a large increase in deepness asymmetry.

The evidence for changing steepness is less uniform, but this lack of uniformity provides a key insight into business cycle steepness that would not be uncovered by examining GDP alone. GDP has been characterized by increasingly steep fluctuations; in the pre-1984 data GDP has a steepness coefficent of -0.16 and this is not statistically different from 0. In the post-1984 data, GDP exhibits a steepness coefficient of -1.11 and this is statistically significant at the 10% level. The insight highlighted here is that this increased steepness of the business cycle is not evident in consumption data, where steepness has fallen at the aggregate level. This suggests that the driving force behind the increased steepness of the business cycle comes from investment. Both non-residential investment and residential investment show large increases in steepness since the onset of the Great Moderation. These steepness coefficients are statistically significant at the 1% level.

While the pattern of sample skewness coefficients is suggestive of increasingly deep and steep cycles in GDP, consumption, and investment, I formally test for differences in the pre- and post-1984 periods. Table 3 reports hypothesis tests for differences in deepness and steepness from pre-1984 to post-1984.

Table 2: Coefficient of skewness test

	Sample	Deepness	P-value	Steepness	P-value
GDP	1949.3-2018.2	-0.40	0.01	-0.06	0.41
	1949.3-1983.4	-0.09	0.35	-0.16	0.24
	1984.1-2007.4	-0.37	0.08	-0.33	0.19
	1984.1-2018.2	-0.71	0.05	-1.11	0.10
Consumption	1949.3-2018.2	-0.49	0.00	-0.38	0.20
	1949.3-1983.4	-0.30	0.06	-0.54	0.13
	1984.1-2007.4	-0.42	0.10	-0.22	0.28
	1984.1-2018.2	-0.68	0.03	-0.39	0.12
Durable Consumption	1949.3-2018.2	-0.54	0.00	-0.12	0.41
	1949.3-1983.4	-0.30	0.11	-0.01	0.49
	1984.1-2007.4	-0.43	0.08	-0.15	0.30
	1984.1-2018.2	-0.77	0.01	-0.48	0.09
Non-durable/Services Consumption	1949.3-2018.2	-0.41	0.01	-0.09	0.32
	1949.3-1983.4	-0.16	0.21	-0.39	0.04
	1984.1-2007.4	-0.44	0.02	-0.31	0.23
	1984.1-2018.2	-0.68	0.02	-0.33	0.10
Gross Private Investment	1949.3-2018.2	-0.65	0.02	-0.19	0.31
	1949.3-1983.4	-0.33	0.08	-0.14	0.34
	1984.1-2007.4	-0.12	0.32	0.07	0.40
	1984.1-2018.2	-1.01	0.05	-0.68	0.17
Non-residential Investment	1949.3-2018.2	-0.42	0.01	-0.56	0.04
	1949.3-1983.4	-0.10	0.30	-0.32	0.13
	1984.1-2007.4	-0.53	0.02	-0.51	0.00
	1984.1-2018.2	-0.75	0.01	-1.30	0.07
Residential Investment	1949.3-2018.2	-0.51	0.00	-0.44	0.09
	1949.3-1983.4	0.01	0.49	-0.33	0.15
	1984.1-2007.4	-0.36	0.18	-0.85	0.01
	1984.1-2018.2	-1.14	0.01	-1.14	0.00

Note: P-values reported for a one-sided hypothesis test of H_0 : Deepness/Steepness ≥ 0 and H_A : Deepness/Steepness < 0. P-values are computed using the Bai and Ng (2005) correction which is described further in the appendix.

Table 3: Changing Business Cycle Asymmetry

	Deepness	P-value	Decision	Steepness	P-value	Decision
GDP	1.419	0.040	Reject	2.422	0.002	Reject
Consumption	0.769	0.121	Do Not Reject	-0.452	0.605	Do Not Reject
Durable Consumption	1.077	0.096	Reject	1.664	0.159	Do Not Reject
Non-durable/Services Consumption	1.025	0.059	Reject	-0.114	0.539	Do Not Reject
Gross Private Investment	1.736	0.019	Reject	2.018	0.086	Reject
Non-residential Investment	1.244	0.026	Reject	3.129	0.013	Reject
Residential Investment	2.391	0.008	Reject	2.344	0.007	Reject

Note: The hypothesis test is $H_0 : \hat{\tau}_1 - \hat{\tau}_2 < 0$ and $H_A : \hat{\tau}_1 - \hat{\tau}_2 \geq 0$. The test is comparing deepness and steepness from 1949:4-1983:4 to 1984:1-2018:2. P-values are bootstrapped using a moving-block bootstrap with 9999 replications. The null hypothesis was rejected for P-values < 0.1.

The evidence for changing business cycle deepness is clear. GDP, consumption (excluding aggregate), and investment have been characterized by increasingly deep fluctuations since the onset of the Great Moderation. The evidence for steepness highlights that GDP experienced a significant change in steepness over the business cycle, and this increased steepness is largely driven by the change in steepness in investment. There is little evidence that steepness in consumption has changed dramatically over the business cycle since the onset of the Great Moderation.

3.1 Sensitivity

In this section I show that the results on the changes in deepness and steepness over the business cycle in GDP, consumption, and investment are robust to various filters, definitions of growth rates, and alternative dates for the onset of the Great Moderation. Specifically, I test for changes in deepness and steepness in the following ways:

1. **Alternative filters**. The baseline trend cycle decomposition for deepness in the paper uses the Hamilton (2017) filter. Table 8 presents hypothesis tests for difference in deepness

from pre- and post-1984 using the Beveridge and Nelson (1981) decomposition (henceforth BN) and Hodrick and Prescott (1980, 1997) (henceforth HP) filter. The evidence using the BN filter is nearly identical with the baseline case, while the HP filter suggests much less evidence for changes in deepness. But as Psaradakis and Sola (2003) highlight, the HP filter struggles to detect cyclical asymmetry relative to the BN decomposition so it is not surprising to get largely different results between the filters.

2. **Alternative growth rate measures**. Coefficients of skewness for steepness were computed using quarter over quarter growth rates. But such growth rates tend to overemphasize high frequency fluctuations and are often not what researchers typically think of as the business cycle. Table 9 reports hypothesis test results for changing steepness pre- and post-1984. The conclusions are nearly identical to those reached under quarter over quarter growth rates, however there is more evidence that changes in steepness are present in the consumption data as well.

3. **Changing volatility**. The onset of the Great Moderation coincided with declining macroeconomic volatility among other changes. To ensure that the changes in deepness and steepness are not driven by changes to the standard deviation, Table 10 reports the third central moment (the numerator in deepness and steepness coefficients) for each variable across the subsamples. The pattern highlights that deepness results are driven by changes in asymmetry, not macroeconomic volatility. For steepness, GDP does not experience a large change in its third central moment which suggests that it is driven more by the change in the volatility.

4. **Alternative onset of the Great Moderation**. The baseline results in the paper use 1984:I as the beginning of the Great Moderation. Table 11 reports hypothesis tests for changing business cycle asymmetry using the onset of the Great Moderation to be 1985:I. The results are nearly identical to the baseline results.

The change in deepness and steepness of US business cycle fluctuations since the onset of the Great Moderation point to empirically important changes in the behavior of the macroeconomy. Understanding the fundamental causes of these changes is an important research question. One promising direction is the rise in rapid accumulation of credit by US households as documented by Jordà et al. (2016).

4 Models With Occasionally Binding Constraints

In this section I present a simple partial equilibrium model with occasionally binding collateral constraints where the only source of uncertainty is a stochastic process for the price of housing. The purpose of the model is not to provide a complete characterization of output or consumption behavior over the business cycle, but instead to show that models with occasionally binding collateral constraints generate a specific form of asymmetry, deepness. However, even in this simple model the asymmetry generated is consistent with the empirical evidence on deepness in consumption.[11]

Finally, I present evidence from a larger scale general equilibrium model via Guerrieri and Iacoviello (2017) where impatient households are occasionally bound by a collateral constraint tied to housing. This model generates asymmetry consistent with the deepness of consumption presented in 3, but does not match the asymmetry of investment or output.

4.1 A Partial Equilibrium Model With Housing As Collateral

In this simple model a representative agent maximizes the following utility function,

$$\max\ E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log(C_t) + \epsilon_H \log(H_t) \right\}$$

where C is consumption of non-durable goods and services and H is consumption of housing. β is the discount factor and ϵ_H is the weight of housing consumption in the utility function, respectively. The agent maximizes utility subject to the following budget constraint,

$$Y_t + H_{t-1}Q_t + B_t = C_t + RB_{t-1} + H_tQ_t \qquad \text{(Budget constraint)}$$

In the model I treat Y as fixed and known, B_t denotes current period borrowing, and R is the exogenous gross real interest rate. Q is the price of housing and is guided by the following exogenous AR(1) process,

[11] Morley and Piger (2012) present empirical evidence that support the use of nonlinear time series models. Occasionally binding constraints introduce non-linearity into DSGE models.

$$\log(Q_t) = \rho_q \log(Q_{t-1}) + u_{q,t}$$
$$u_{q,t} \sim N(0, \sigma_q^2)$$

Lastly, the representative agent is subject to a collateral constraint bounding borrowing from exceeding a fraction s of the present value of next periods housing wealth,

$$B_t \leq s\frac{E_t\{Q_{t+1}\}H_t}{R} \qquad \text{(Collateral constraint)}$$

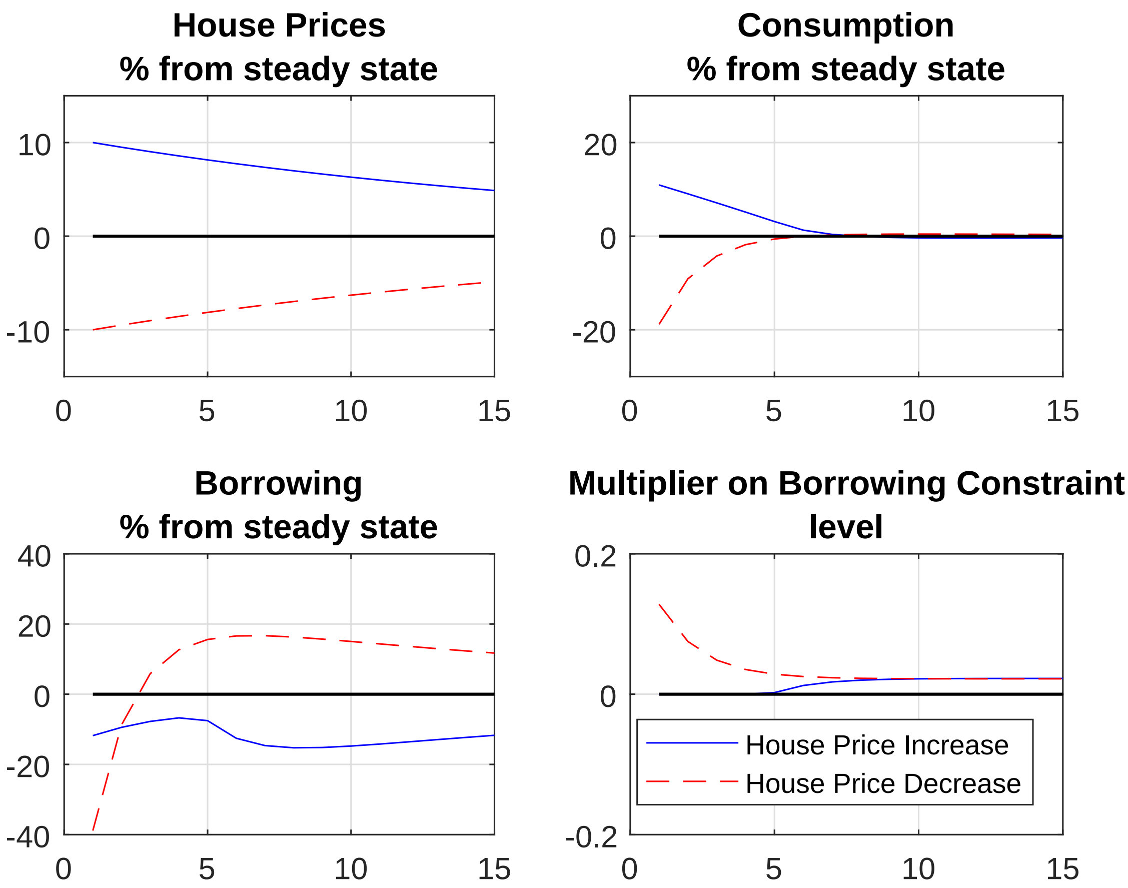
This borrowing constraint yields a complementary slackness condition which states that μ_t, the borrowing constraint multiplier, is strictly greater than zero when the constraint is binding and zero when the agent is unconstrained,

$$\mu_t\left(s\frac{E_t\{Q_{t+1}\}H_t}{R} - B_t\right) = 0 \qquad \text{(C.S. condition)}$$

To pin down the steady state of the model it is assumed that the representative agent is sufficiently impatient such that the agent prefers consumption today relative to consumption in the future. This implies that $R\beta < 1$ and μ, the constraint on borrowing, is strictly greater than 0 in the steady state. To gain some insight into the type of asymmetry generated by the model I calibrate and simulate the model using the following parameter values: $R = 1.02$, $\beta = 0.96$, $\rho_q = 0.95$, $s = 0.9$ and $\epsilon_H = 0.0827$.[12] Figure 2 presents impulse responses to a 10% positive and negative house price shock.

[12]To solve the model I use the *dynareOBC toolkit* provided by Holden (2016). Solving the model using the *Occbin* framework from Guerrieri and Iacoviello (2015) yields nearly identical results.

Figure 2: IRF to a 10% housing price shock



Note: Impulse responses are computed using a 10% shock to house prices.

The response of consumption to equally sized housing price shocks illustrates the fundamental asymmetry in the model. Since the agent's borrowing constraint is binding in the steady state, a negative house price shock causes consumption to fall greater than one for one. This response is driven by two channels: the first is that a fall in housing leads to a wealth effect, the second, is a debt de-leveraging effect. Since the borrowing constraint is binding, any downward movements in house prices require that the agent pay down some of their existing debt. These two effects in combination lead to the greater than one for one response of consumption to downward house price movements.

In contrast, a positive shock to house prices leads to a roughly one for one response in consumption. This asymmetric response is driven by the fact that the borrowing multiplier reaches the lower bound, staying there for 5 periods (as depicted in the bottom right quadrant of the IRF). The borrowing constraint being 0 implies that $B_t < s\frac{E_t\{Q_{t+1}\}H_t}{R}$, the agent's borrowing constraint is not binding. This non-binding versus binding response leads to the asymmetric response of consumption, but the key insight is that this asymmetry is a level asymmetry consistent with deepness as described by Sichel (1993). Consumption falls more in response to the same shock than it rises in response to the positive shock.

To reinforce this point, I simulate the model setting $\sigma_q = 0.0175$ to match the standard deviation of the growth rate of house prices in the data (Guerrieri and Iacoviello 2017). Table 4 reports deepness and steepness coefficients,

Table 4: Deepness and steepness from the model

	Case I: $\sigma_q = 0.0175$	
	Deepness	*Steepness*
House Prices	-0.03	0.018
Consumption	-0.241	0.021

Note: The model is simulated for 11,000 periods and the first 1,000 periods are dropped to avoid any issues with initial conditions. Before computing deepness the simulated data is passed through the Hamilton (2017) filter, but this no impact on the result. Under this calibration the borrowing constraint is binding roughly 15% of the time.

The simulation offers results consistent with the intuition provided for the impulse response functions. The main type of asymmetry generated by the collateral channel in this model is deepness, a level asymmetry. Symmetric house price movements lead to asymmetric movements in consumption.

4.2 General Equilibrium

The partial equilibrium model provides insights into the fundamental type of asymmetry generated from occasionally binding collateral constraints, but whether this intuition would hold when expanded to a larger general equilibrium model with a variety of shocks and frictions is not obvious. To reinforce that this intuition does hold, I simulate evidence from Guerrieri and Iacoviello (2017) (henceforth GI). The GI model is an estimated two agent new Keynesian

(TANK) model where impatient households are subject to an occasionally binding collateral constraint and there is a zero lower bound (ZLB) on nominal interest rates set by the central bank. The model contains a variety of frictions including habit formation in consumption and housing, investment adjustment costs, and sticky prices and wages. The model contains 6 different shocks which include housing demand shocks, investment specific technology shocks, price and wage markup shocks, monetary policy shocks, and an intertemporal preference shock. To isolate the impact of occasionally binding collateral constraints on asymmetry, Table 5 reports deepness and steepness in the model for a case where the ZLB is imposed and a case where it is not imposed.

Table 5: Deepness and Steepness from Guerrieri and Iacoviello (2017)

	Case I: Two OBCs: Collateral constraint & ZLB	
	Deepness	*Steepness*
Output	-0.112	-0.014
Aggregate Consumption	-0.156	-0.059
Patient Consumption	-0.017	-0.024
Impatient Consumption	-0.523	-0.114
Investment	-0.066	-0.005
	Case II: One OBC: Collateral constraint	
	Deepness	*Steepness*
Output	-0.106	-0.015
Aggregate Consumption	-0.155	-0.059
Patient Consumption	-0.017	-0.024
Impatient Consumption	-0.520	-0.112
Investment	-0.005	-0.005

Note: The model is simulated for 11,000 periods and the first 1,000 periods are dropped to avoid any issues with initial conditions. Before computing deepness the simulated data is passed through the Hamilton (2017) filter, but this has no impact on the results. The calibration used in the simulation of the model is the baseline calibration presented in Guerrieri and Iacoviello (2017).

Deepness and steepness from GI indicate that the conclusions reached from the partial equilibrium model do indeed hold in the general equilibrium framework. The model generates deepness in impatient consumption, but only moderately deep consumption cycles at the aggregate level. In addition, the deepness in investment and output are counterfactual to the data. Likewise, the model does not inherently generate steepness which is also counterfactual to the data.

5 Conclusion

The empirical evidence presented in this paper documents significant deepness in output, consumption, and investment. Deepness has increased in all of these variables since the onset of the Great Moderation. Turning to steepness, output has exhibited significant increases in the steepness of its cycle, however the evidence herein suggests that this is driven by changes in the fluctuations of investment over the business cycle. These results are robust to a variety of trend cycle decompositions, growth rate measures, and definitions of the onset of the Great Moderation.

In the second half of this paper I highlighted the main asymmetry generated from models featuring occasionally binding collateral constraints is deepness. The paper presented a simple partial equilibrium model where housing serves as collateral in the presence of disturbances to house prices. This simple model generated asymmetric consumption responses to house price shocks. To verify this asymmetry is present in larger scale DSGE models, I presented simulation evidence from Guerrieri and Iacoviello (2017). This model also generated deepness in consumption, however the model did not match the asymmetries in output or investment which were documented in the empirical section.

Documenting asymmetry and its changing nature is important to business cycle research which relies heavily on stylized facts about fluctuations. By definition linearized models with symmetric disturbances cannot capture asymmetry. The collateral channel offers one mechanism for generating deepness, however these mechanisms fail to match the steep present in the output and investment data. A model that can explain both types of asymmetry and account for the changing nature of these asymmetries is an open and important research avenue.

References

Atolia, M., Gibson, J. and Marquis, M.: 2018, Asymmetry and the amplitude of business cycle fluctuations: A quantitative investigation of the role of financial frictions, *Macroeconomic Dynamics* **22**(2), 279–306.

Bai, J. and Ng, S.: 2005, Tests for skewness, kurtosis, and normality for time series data, *Journal of Business & Economic Statistics* **23**, 49–60.

Beveridge, S. and Nelson, C.: 1981, A new approach to the decomposition of economic time series into permanent and transitory components with particular attention to the measurement of the 'business cycle', *Journal of Monetary Economics* pp. 151–174.

Brault, J. and Khan, H.: 2018, The Shifts in Lead-Lag Properties of the US Business Cycle, *Carleton Economic Papers 18-03*.

Bry, G. and Boschan, C.: 1971, *Cyclical Analysis of Time Series: Selected Procedures and Computer Programs*, National Bureau of Economic Research, Inc.

Fatàs, A. and Mihov, I.: 2013, Recoveries, *CEPR Discussion Papers 9551*, C.E.P.R. Discussion Papers.

Ferraro, D.: 2018, The asymmetric cyclical behavior of the u.s. labor market, *Review of Economic Dynamics* **30**, 145 – 162.

Garin, J., Pries, M. J. and Sims, E. R.: 2018, The relative importance of aggregate and sectoral shocks and the changing nature of economic fluctuations, *American Economic Journal: Macroeconomics* **10**(1), 119–148.

Guerrieri, L. and Iacoviello, M.: 2015, OccBin: A toolkit for solving dynamic models with occasionally binding constraints easily, *Journal of Monetary Economics* **70**(C), 22–38.

Guerrieri, L. and Iacoviello, M.: 2017, Collateral constraints and macroeconomic asymmetries, *Journal of Monetary Economics* **90**(C), 28–49.

Hamilton, J. D.: 2017, Why you should never use the Hodrick-Prescott filter, *Review of Economics and Statistics* p. Forthcoming.

Hansen, G. and Prescott, E.: 2005, Capacity constraints, asymmetries, and the business cycle, *Review of Economic Dynamics* **8**(4), 850 – 865.

Harding, D. and Pagan, A.: 2002, Dissecting the cycle: a methodological investigation, *Journal of Monetary Economics* **49**(2), 365–381.

Hodrick, R. J. and Prescott, E. C.: 1980, Postwar U.S. business cycles: An empirical investigation, *Technical report*, Carnegie Mellon University.

Hodrick, R. J. and Prescott, E. C.: 1997, Postwar U.S. business cycles: An empirical investigation, *Journal of Money, Credit and Banking* **29**(1), 1–16.

Holden, T.: 2016, Computation of solutions to dynamic models with occasionally binding constraints, *Econstor preprints*, ZBW - Leibniz Information Centre for Economics.

Jensen, H., Petrella, I., Ravn, S. H. and Santoro, E.: 2017, Leverage and Deepening Business Cycle Skewness, *CEPR Discussion Papers 12239*, C.E.P.R. Discussion Papers.

Jordà, Ò., Schularick, M. and Taylor, A. M.: 2016, Macrofinancial History and the New Business Cycle Facts, *NBER Macroeconomics Annual 2016, Volume 31*, NBER Chapters, National Bureau of Economic Research, Inc, pp. 213–263.

McConnell, M. M. and Perez-Quiros, G.: 2000, Output fluctuations in the united states: What has changed since the early 1980's?, *American Economic Review* **90**(5), 1464–1476.

McKay, A. and Reis, R.: 2006, The Brevity and Violence of Contractions and Expansions, *NBER Working Papers 12400*, National Bureau of Economic Research, Inc.

Mitchell, W. C.: 1927, Business Cycles: The Problem and Its Setting, *Technical report*, National Bureau of Economic Research, Inc.

Morley, J. and Piger, J.: 2012, The Asymmetric Business Cycle, *The Review of Economics and Statistics* **94**(1), 208–221.

Popov, A.: 2014, Credit constraints, equity market liberalization, and growth rate asymmetry, *Journal of Development Economics* **107**(1), 202–214.

Psaradakis, Z. and Sola, M.: 2003, On detrending and cyclical asymmetry, *Journal of Applied Econometrics* **18**(3), 271–289.

Razzak, W.: 2001, Business Cycle Asymmetries: International Evidence, *Review of Economic Dynamics* **4**(1), 230–243.

Sichel, D.: 1993, Business cycle asymmetry: A deeper look, *Economic Inquiry* **31**(2), 224–236.

Verbrugge, R.: 1997, Investigating cyclical asymmetries, *Studies in Non-linear Dynamics & Econometrics* **2**(1), Article 2.

Appendix

Data Sources

The data used in this paper were obtained from the Federal Reserve Economic Data database (FRED) maintained by the Federal Reserve Bank of St. Louis. The data were obtained on December 8, 2018. Nominal variables are converted to real using each series corresponding deflator. The series used in the paper and their corresponding FRED codes are described below:

Table 6: Data sources

Variable	**Sample**	**FRED code**
GDP	1947:I-2018:II	GDP
GDP Deflator	1947:I-2018:II	GDPDEF
Personal Consumption Expenditures (PCE)	1947:I-2018:II	PCEC
PCE Implicit Price Deflator	1947:I-2018:II	DPCERD3Q086SBEA
PCE: Durable Goods	1947:I-2018:II	PCDG
PCE: Durable Goods Deflator	1947:I-2018:II	DDURRD3Q086SBEA
PCE: Nondurable Goods	1947:I-2018:II	PCND
PCE: Nondurable Goods Deflator	1947:I-2018:II	DNDGRD3Q086SBEA
PCE: Services	1947:I-2018:II	PCESV
PCE: Services Deflator	1947:I-2018:II	DSERRD3Q086SBEA
Gross Private Investment	1947:I-2018:II	GPDI
Gross Private Investment Deflator	1947:I-2018:II	A006RD3Q086SBEA
Residential Investment	1947:I-2018:II	PRFI
Residential Investment Deflator	1947:I-2018:II	A011RD3Q086SBEA
Non-residential Investment	1947:I-2018:II	PNFI
Non-residential Investment Deflator	1947:I-2018:II	A008RD3Q086SBEA

Non-durable consumption and services are added together after deflating each series with its respective price deflator.

Bai and Ng (2005) Standard Errors for Sample Skewness

To compute standard errors for skewness coefficients in the paper I use the correction suggested by Bai and Ng (2005). Under *Corollary 1*, when testing for $\tau = 0$, one only needs to test the sampling properties of,

$$\frac{1}{\sqrt{T}} \sum_{t=1}^{T} \mathbf{Z}_t = \frac{1}{\sqrt{T}} \sum_{t=1}^{T} \begin{bmatrix} (X_t - \mu)^3 \\ (X_t - \mu) \end{bmatrix}$$

where $\mathbf{\Gamma} = \lim_{T \to \infty} \mathrm{E}(\mathbf{Z}\mathbf{Z}')$. $\mathbf{\Gamma}$ is estimated using a Newey-West approximation where the number of lags is chosen according to the rule-of-thumb $T^{1/4}$. This leads to the following result,

$$\sqrt{T}\hat{\tau} \to \mathrm{N}\left(0, \frac{\alpha_2 \mathbf{\Gamma}_{22} {\alpha'}_2}{\sigma^6}\right)$$

where $\alpha_2 = [1, -3\sigma^2]$. Then the empirical standard error is given by $s(\hat{\tau}) = \left(\frac{\alpha_2 \mathbf{\Gamma}_{22} {\alpha'}_2}{\sigma^6}\right)^{1/2}$. The difference between this standard error and the typical sample skewness standard error is the normalizing constant in the denominator.

Additional Results

Table 7: Ljung-Box test results

	Hamilton Filtered		**Growth Rates**	
	Q-stat	P-value	Q-stat	P-value
GDP	602.57	0.00	57.05	0.00
Consumption	614.71	0.00	42.19	0.00
Durable Consumption	609.37	0.00	21.21	0.02
Non-durable and Services	565.48	0.00	116.09	0.00
Gross Private Investment	645.51	0.00	36.44	0.00
Fixed Investment	633.60	0.00	112.44	0.00
Non-residential Investment	602.05	0.00	104.25	0.00
Residential Investment	639.28	0.00	114.59	0.00

Note: Ljung-Box test results were performed allowing for autocorrelation up to lag 10.

Table 8: Changing Business Cycle Asymmetry: alternative filters

	Deepness (HP)	P-value	Decision	Deepness (BN)	P-value	Decision
GDP	-0.487	0.775	Do Not Reject	1.036	0.121	Do Not Reject
Consumption	0.677	0.145	Do Not Reject	1.029	0.085	Reject
Durable Consumption	1.985	0.058	Reject	3.134	0.025	Reject
Non-durable/Services Consumption	-0.108	0.639	Do Not Reject	0.269	0.287	Do Not Reject
Gross Private Investment	2.114	0.035	Reject	2.757	0.079	Reject
Non-residential Investment	-0.054	0.530	Do Not Reject	1.403	0.035	Reject
Residential Investment	-0.036	0.555	Do Not Reject	2.465	0.011	Reject

Note: The hypothesis test is $H_0 : \hat{\tau}_1 - \hat{\tau}_2 < 0$ and $H_A : \hat{\tau}_1 - \hat{\tau}_2 \geq 0$. The test is comparing deepness and steepness from 1949:4-1983:4 to 1984:1-2018:2. P-values are bootstrapped using a moving-block bootstrap with 9999 replications.

Table 9: Changing Business Cycle Asymmetry: alternative growth rates

	Steepness (MA)	P-value	Decision	Steepness (YoY)	P-value	Decision
GDP	1.765	0.092	Reject	1.799	0.110	Do Not Reject
Consumption	1.161	0.049	Reject	1.144	0.052	Reject
Durable Consumption	2.975	0.008	Reject	2.533	0.019	Reject
Non-durable/Services Consumption	0.465	0.174	Do Not Reject	0.452	0.181	Do Not Reject
Gross Private Investment	3.727	0.072	Reject	5.015	0.107	Do Not Reject
Non-residential Investment	1.847	0.029	Reject	1.550	0.039	Reject
Residential Investment	2.704	0.003	Reject	3.537	0.003	Reject

Note: The hypothesis test is $H_0 : \hat{\tau}_1 - \hat{\tau}_2 < 0$ and $H_A : \hat{\tau}_1 - \hat{\tau}_2 \geq 0$. The test is comparing deepness and steepness from 1949:4-1983:4 to 1984:1-2018:2. P-values are bootstrapped using a moving-block bootstrap with 9999 replications.

Table 10: 3rd central moments: Pre- and post-1984

	Sample	Deepness	Steepness
GDP	1949.3-2018.2	-14.51	-0.05
	1949.3-1983.4	-6.11	-0.29
	1984.1-2007.4	-4.87	-0.05
	1984.1-2018.2	-14.42	-0.23
Consumption	1949.3-2018.2	-10.84	-0.21
	1949.3-1983.4	-8.79	-0.57
	1984.1-2007.4	-3.33	-0.02
	1984.1-2018.2	-11.13	-0.05
Durable Consumption	1949.3-2018.2	-541.82	-5.54
	1949.3-1983.4	-422.16	-2.47
	1984.1-2007.4	-226.53	-1.74
	1984.1-2018.2	-527.37	-6.30
Non-durable/Services Consumption	1949.3-2018.2	-2.67	-0.01
	1949.3-1983.4	-1.41	-0.06
	1984.1-2007.4	-0.97	-0.01
	1984.1-2018.2	-3.82	-0.02
Gross Private Investment	1949.3-2018.2	-1407.72	-17.18
	1949.3-1983.4	-639.57	-25.83
	1984.1-2007.4	-210.15	-2.81
	1984.1-2018.2	-2171.47	-22.13
Non-residential Investment	1949.3-2018.2	-369.58	-6.82
	1949.3-1983.4	-115.40	-6.58
	1984.1-2007.4	-362.00	-2.35
	1984.1-2018.2	-640.94	-8.67
Residential Investment	1949.3-2018.2	-3627.71	-44.14
	1949.3-1983.4	912.05	-60.75
	1984.1-2007.4	-1735.93	-19.10
	1984.1-2018.2	-6856.03	-40.54

Note: P-values reported for a one-sided hypothesis test of $H_0 : D(c)/S(c) \geq 0$ and $H_A : D(c)/S(c) < 0$.

Table 11: Changing Business Cycle Asymmetry: Pre- and Post-1985

	Deepness	P-value	Decision	Steepness	P-value	Decision
GDP	1.629	0.021	Reject	2.715	0.001	Reject
Consumption	0.822	0.122	Do Not Reject	-0.584	0.645	Do Not Reject
Durable Consumption	1.351	0.062	Reject	1.501	0.185	Do Not Reject
Non-durable/Services Consumption	0.960	0.071	Reject	-0.148	0.559	Do Not Reject
Gross Private Investment	2.122	0.007	Reject	2.604	0.056	Reject
Non-residential Investment	1.134	0.038	Reject	3.199	0.012	Reject
Residential Investment	3.432	0.003	Reject	2.279	0.012	Reject

Note: The hypothesis test is $H_0 : \hat{\tau}_1 - \hat{\tau}_2 < 0$ and $H_A : \hat{\tau}_1 - \hat{\tau}_2 \geq 0$. The test is comparing deepness and steepness from 1949:4-1984:4 to 1985:1-2018:2. P-values are bootstrapped using a moving-block bootstrap with 9999 replications.